Study of a Spring-Damper for Boat Chair Suspension

ir. J.G. Gruijters

June the 27th, 2020

Abstract

This study is made to use analytic simulation methods to study and compare different designs for the suspension of a chair used on a sea-going vessel. This article shows the simulation of several designs and their responses to similar vessel motions. The mechanical springs and gas springs have a trade-off between comfort and maximum operational conditions. The active system has the greatest comfort, but offers the least protection in rougher weather. Another important parameter that are not simulated but have a huge impact is enlarging the stroke (larger stroke means higher operational range).

Sea-going small and high speed vessels have chairs which are suspended individually. This suspension is not only to make the ride more comfortable during nice weather, but is also meant to prevent injuries when the weather is not so nice.

The seat can be either suspended by a mechanical or a gas spring. The springs must also contain some damping, to dissipate the energy from the movements.

In this article both types and some special types will be shown and simulated to see how well they perform. The damping will be kept equal throughout the article, to show clearly the differences between the designs, and not the influence of higher or lower damping.

1 The model

To start the study, one general model will be used on which all simulations are based. A graphic representation of the model is shown in Figure 1.

The top part is a standard mass-spring-damper model as often used in theoretical studies. The connection to the world is used to excite the system, similar to how the vessel will be moving. The forces of the spring and damper are thus not only dependent on the position and speed of the mass, but also of the vessel compared to the world. Therefore, the general equation of motion for the passenger on the seat is found:

$$m\,\ddot{z}_2 - d\,(\dot{z}_1 - \dot{z}_2) - c\,(z_1 - z_2) + 600 \;=\; -m\,g \tag{1}$$

where the +600 at the spring is the pretension of the spring when fully extended (600N $\approx$ 60kg). The other quantities are shown in Table 1

This leads to the following set of first order differential equations when the suspension is not end of stroke:

$$\begin{bmatrix} \ddot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{-m\,g + d\,(\dot{z}_1 - x_1) + c\,(z_1 - x_2) + 600 + c\,0.1359}{m} \\ x_1 \end{bmatrix} \tag{2}$$

Unfortunately, it is possible that the suspension hits the end of stroke position. At that point, the suspension is end of stroke and the position is fixed to the position of the vessel. This can lead to high forces and thus needs to be considered. This means that when $z_2 - z_1 \;>\; s$ and when $z_2 - z_1 \;<\; 0$ the non linearity appears, where the impact force brings the speed back to 0[m/s].

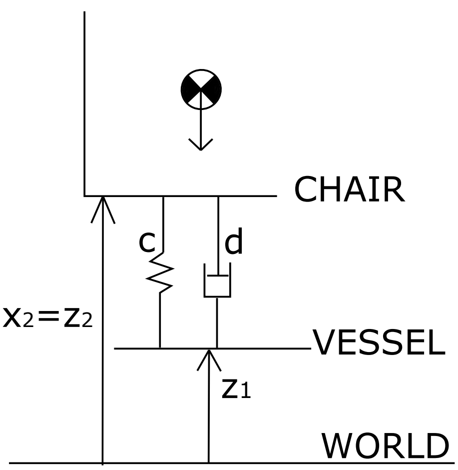

Figure 1: Graphic representation of the mathematical model.

Table 1: Explanation of the quantities

Symbol	quantity	unit
c	Spring stiffness	$[N/m]$
d	Damping coefficient	$[Ns/m]$
g	Gravitational acceleration	$10[m/s^2]$
m	Mass of the passenger + chair	$80[kg]$
$\dot{z}_1$	Velocity of the vessel	$[m/s]$
z_1	Position of the vessel w.r.t. the world	$[m]$
$\dot{z}_2$	Velocity of the passenger and chair	$[m/s]$
z_2	Position of the passenger and chair w.r.t. the world	$[m]$
	Stroke of suspension	0.1359[m]

First the impact force. The impact force is calculated by setting the work done by the impact ($W = F_{impact}\, s$) equal to the kinetic energy ($E_{kin} = \frac{1}{2}\, m\, v^2$):

$$F_{impact}\, s = \frac{1}{2}\, m\, v^2 \tag{3}$$

$$F_{impact} = \frac{m\, v^2}{2\, s} \tag{4}$$

The stroke which can be deformed is assumed to be 1[mm] in these calculations. A small stroke is required to prevent the force of going to infinity.

This means that at the end of stroke the position needs to be forced to the end position (stroke + vessel position z_1) in the simulation. This means that the spring is no longer acting on the mass while the impact force is acting on the mass. The differential equation for extended position is:

$$\begin{bmatrix} \ddot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{\frac{-0.5*m\,(x_1-\dot{z}_1)^2}{0.001} - m\, g + d\,(\dot{z}_1 - x_1)}{m} \\ x_1 \end{bmatrix} \tag{5}$$

For the retracted position the main difference is the direction of the impact force, the mass is now suspended by the steel contact, which leads to the following differential equation:

$$\begin{bmatrix} \ddot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{\frac{0.5*m\,(x_1-\dot{z}_1)^2}{0.001} + d\,(\dot{z}_1 - x_1) + c\,(z_1 - x_2) + 600 + c\, 0.1359}{m} \\ x_1 \end{bmatrix} \tag{6}$$

This is the basic model that will be adapted for each spring damper system. The mass of the passenger and spring will be adapted to each other, meaning that this will not be a great influence.

2 Linear Spring Model

A linear spring means that the spring stiffness is constant over the full stroke. This can be (approximately) achieved with mechanical springs. the used parameters are shown in Table 2. Important to know is that this system has under-damping with these quantities, see [1] for further reading.

The natural frequency is explained in [1], and the natural frequency of the damped system is 0.95[Hz]. The Bode diagram for this system is shown in Figure 2.

2.1 Spring pretension adjusted to weight

In this first subsection, the spring will be compressed by 0.05[m] by the weight of the passenger until the spring force equals the weight of the passenger and the chair.

Table 2: Used quantities for the linear spring

Symbol	quantity	unit
c	Spring stiffness	$4000[N/m]$
d	Damping coefficient	$600[Ns/m]$

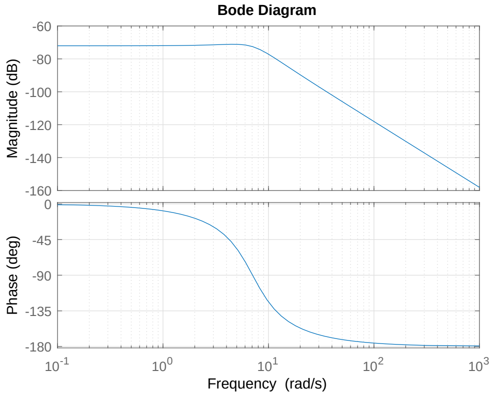

Figure 2: The Bode diagram of the system with a linear spring.

2.1.1 Static results

Not only the normal usage case will be shown for the static results, but also a too heavy passenger and the passenger suddenly standing up is shown for model verification.

For the static results the position of the vessel is kept constant at 0[m] in the world. This means that the system practically becomes a standard mass-spring-damper system, which is easy to verify.

Normal use During normal use, a passenger and seat weight of 80[kg] is simulated. The simulation starts at extended stroke, meaning that the passenger is just sitting down. The results are shown in Figure 3. The eventual position is checked using a simple calculation: pretension is 600[N], the weight of seat and passenger is 800[N], meaning that the compression of the spring should deliver 200[N]. With a spring stiffness of 4000[N/m], the displacement should be 0.05[m]. The stroke of 0.1359[m] minus the displacement means that the stroke should end up at 0.0859[m], which is equal to the result in the simulation.

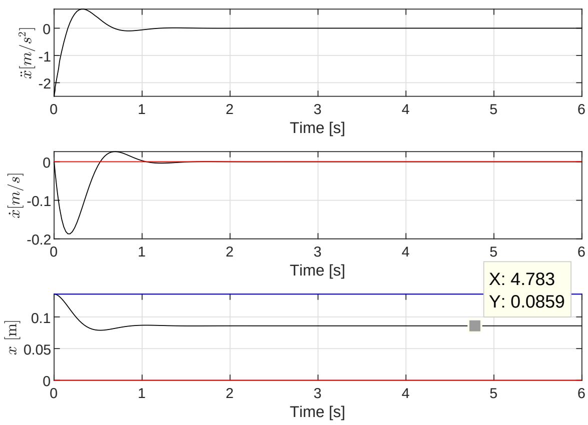

Figure 3: The acceleration, velocity and position for the static case during normal use (sitting down).

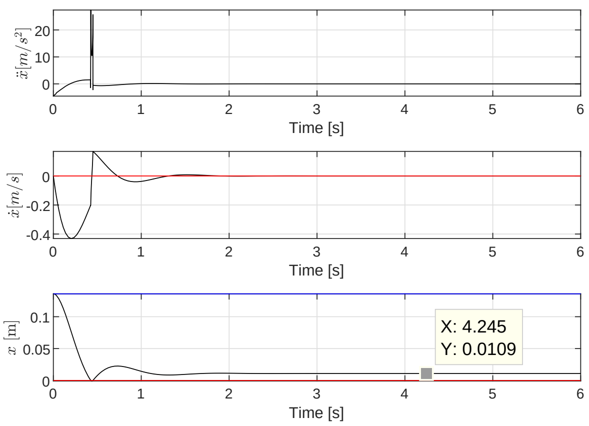

Figure 4: The acceleration, velocity and position for the static case for a too heavy passenger sitting down.

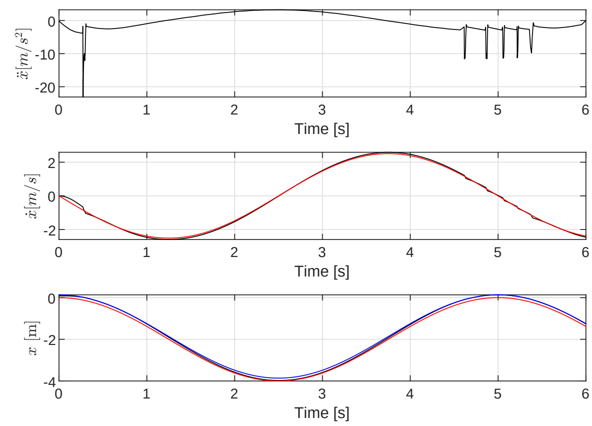

Figure 6: The acceleration, velocity and position for a sinusoidal movement of the vessel with an amplitude of 2[m] and a period time f 5[s].

Too heavy passenger For the too heavy passenger the weight of the passenger and seat is increased to 110[kg]. This 1100[N] minus the 600[N] pretension, the spring needs to deliver an extra 500[N], which means 500/4000=0.125[m]. With the stroke of 0.1359[m] the stroke left is 0.0109[m]. However, due to the dynamics the suspension will hit the end of stroke position. Due to the short deformation allowed for the impact, the force and thus the acceleration at that point will be huge. This is all shown in Figure 4.

Standing up suddenly The same end of stroke position applies to the stroked out position. This happens for instance when the passenger stands up, and only the weight of the chair is left. The weight of the passenger is reset to 80[kg] and the starting position is at the equilibrium position of the passenger sitting down, so 0.0859[m]. After 2[s] the passenger stands up and the suspension extends to the extended position and hits end of stroke. The impact of the end of stroke is shown by a spike in the acceleration. The results are shown in Figure 5. That the stroke is higher than the 0.1359[m] is due to the braking distance.

2.1.2 Results with wave motion

Up till now the vessel was assumed steady, so now the vessel starts moving on the waves. With an amplitude of 2[m] the vessel will move a total of 4[m]. With a period time of up to 6[s] the chair does not hit the end of stroke positions, which is why Figure 6 shows a period time of 5[s] and the acceleration shows the impact forces.

2.1.3 Results for steep drop of the vessel

The steep drop is simulated using a cosine and it simulates a drop of 6[m] within 2[s], and this suddenly stops. This represents a situation that the vessel drops between the waves and afterwards it hits the next wave. The drop is gradual,

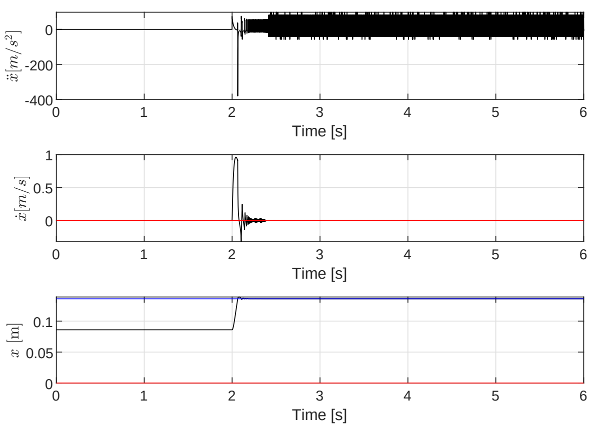

Figure 5: The acceleration, velocity and position for the static case and the passenger suddenly standing up.

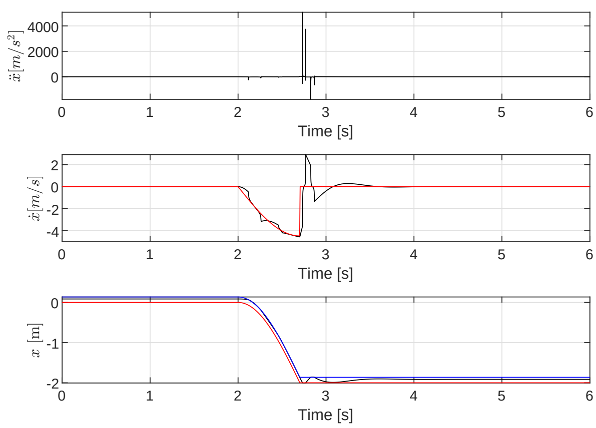

Figure 7: The acceleration, velocity and position for a steep drop of the vessel.

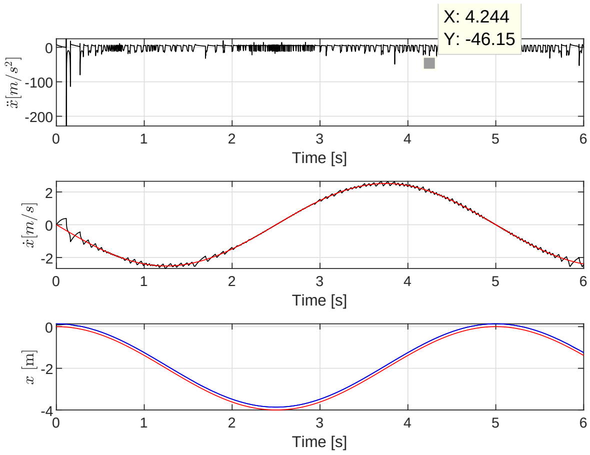

Figure 8: The acceleration, velocity and position for a sinusoidal movement of the vessel with an amplitude of 2[m] and a period time f 5[s] for the higher spring pretension.

as gravity needs to accelerate the vessel, but when the vessel hits the next wave it is not a nice sinusoidal shape, but the position is suddenly at standstill. The spring should soften the shock. The results in Figure 7 show high accelerations, which means there are high forces on the passenger.

2.2 Much stronger pretension

In this subsection the spring pretension is much larger than the weight of the passenger, making sure the chair is at maximum stroke when the passenger sits down.

Therefore, the pretension is increased from 600N to 1200N.

2.2.1 Results with wave motion

For the results of the sinusoidal wave, see Figure 8, the accelerations for the higher spring pretension are higher than for the normal spring. This indicates higher forces on the passenger than for the normal spring pretension. Considering that the steep drop results in accelerations of orders of magnitude higher, it is not yet convincing.

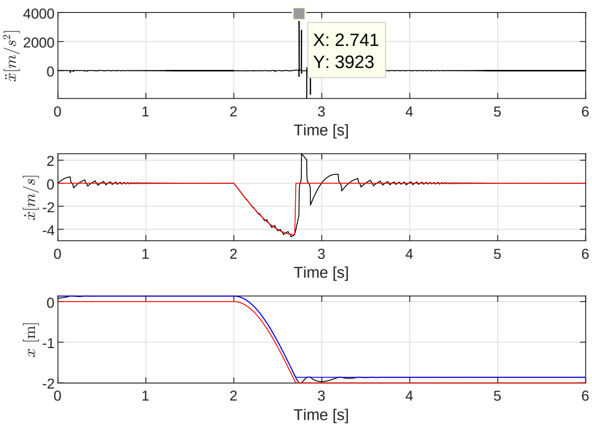

Figure 9: The acceleration, velocity and position for a steep drop of the vessel for the higher pretension.

2.2.2 Results for steep drop of the vessel

The accelerations for the steep drop are slightly lower than for the spring with lower pretension, as shown in Figure 9, although the difference is not that large. But it is an indication that this setup has advantages compared to the spring adjusted to the passenger's weight.

3 Progressive spring

The idea behind the progressive spring is that the spring stiffness increases when it is compressed. This is for instance done by changing the pitch of the spring helical for some part of the spring's length. An example is shown in Figure 10.

Figure 10: An example of a progressive spring.

This means that from a certain spring length Q the spring stiffness changes from c to c_2. The spring force is now not only dependent on the compression, but also a changing spring stiffness. Therefore, two equations are required instead of one in the normal operating range of the spring When the spring is between length Q and extended position (0.1359[m]):

$$\begin{bmatrix} \ddot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{-m\,g + d\,(\dot{z}_1 - x_1) + c\,(z_1 - x_2) + 600 + c\,0.1359}{m} \\ \dot{z}_2 \end{bmatrix} \tag{7}$$

When the spring is between length Q and retracted position:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{-m\,g + d\,(\dot{z}_1 - x_1) + c_2\,(z_1 - x_2) - c\,Q + 600 + c\,0.1359 + c_2\,Q}{m} \\ \dot{z}_2 \end{bmatrix} \tag{8}$$

The forces at the extended position do not change, as the spring force is internally blocked to act on the mass:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{\frac{-0.5*m\,(x_1 - \dot{z}_1)^2}{0.001} - m\,g + d\,(\dot{z}_1 - x_1)}{m} \\ x_1 \end{bmatrix} \tag{9}$$

For the retracted position the spring force is updated to the stiffer stiffness:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{\frac{0.5*m\,(x_1-\dot{z}_1)^2}{0.001}+d\,(\dot{z}_1-x_1)+c_2\,(z_1-x_2)-c\,Q+600+c\,0.1359+c_2\,Q}{m} \\ x_1 \end{bmatrix} \tag{10}$$

3.1 Spring adjusted to passengers weight

Similar as for the linear spring, first a spring is used that is dependent on the passenger's weight, to give a proper suspension. All parameters are kept the same in comparison with the linear spring. The extra parameters required for this simulation are shown in Table 3.

3.1.1 Results with wave motion

The wave motion simulation is the same as done for the linear spring. The results for the wave motion are shown in Figure 12. The accelerations, especially around 5[s] in the simulation, of the progressive spring are higher, indicating that the impact forces were higher as well. In that sense the progressive spring is worse than a linear spring.

3.1.2 Results for steep drop of the vessel

The steep drop simulation is similar as done before for the linear spring. The result of the simulation is shown in Figure 13. The resulting accelerations and thus forces are the similar to the linear spring, mainly due to the same velocity change and hitting the end of stroke position. Increasing the spring stiffness leads thus to higher accelerations.

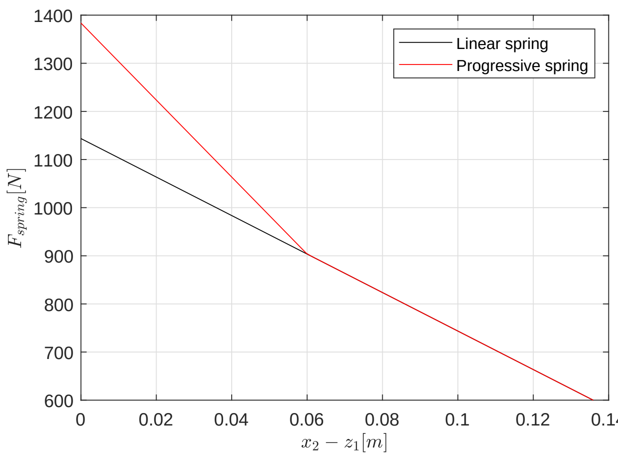

Figure 11: The difference between spring force of a linear and progressive spring.

Table 3: Used quantities for the progressive spring

Symbol	quantity	unit
c_2	Spring stiffness	$8000[N/m]$
Q	Point of stiffness change	$0.06[m]$

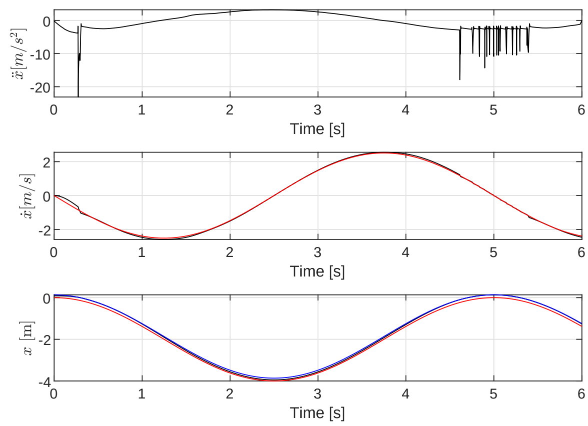

Figure 12: The acceleration, velocity and position for a sinusoidal movement of the vessel with an amplitude of 2[m] and a period time f 5[s].

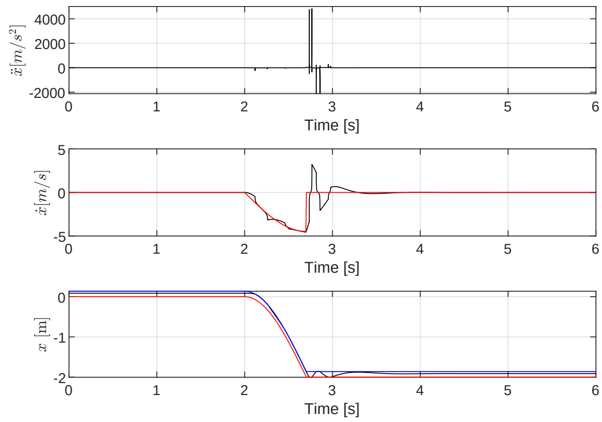

Figure 13: The acceleration, velocity and position for a steep drop of the vessel.

3.2 Stronger pretension of the spring

Similar as done for the linear spring, the pretension of the spring can also be increased to above the passenger's weight. This means that the suspension is normally always extended, unless is has to dampen some shocks by the waves. The pretension at fully extended stroke is now increased to 1200[N], the same as for the linear spring.

3.2.1 Results with wave motion

The results for the wave function are shown in Figure 14. It is clearly visible that these accelerations are larger than for the spring adjusted to the weight, although the accelerations for the steep drop are much larger.

3.2.2 Results for steep drop of the vessel

The results for the steep drop are shown in Figure 15. This shows that the maximum accelerations are almost a factor

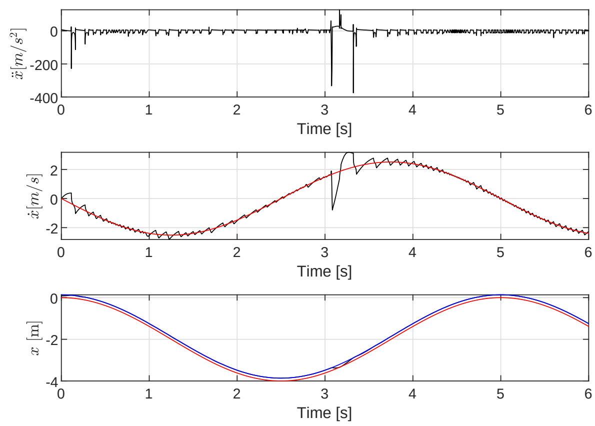

Figure 14: The acceleration, velocity and position for a sinusoidal movement of the vessel with an amplitude of 2[m] and a period time f 5[s] with higher spring pretension.

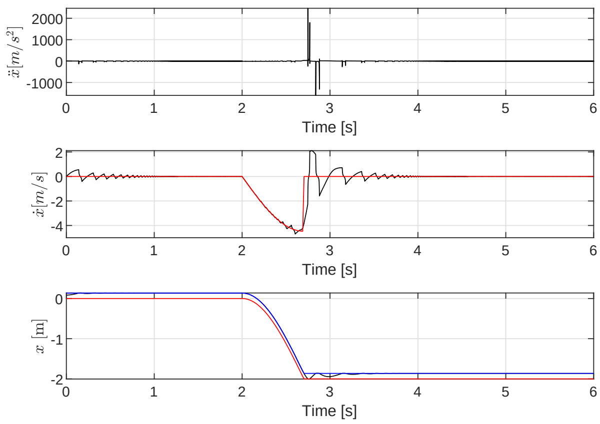

Figure 15: The acceleration, velocity and position for a steep drop of the vessel with higher spring pretension.

two lower than the original accelerations, meaning that the impact (and thus mainly the speed of the impact) is lower. This means that this type of spring has an advantage when used in this manner.

3.3 Conclusion

The progressive spring has an advantage over the linear spring for large impacts (steep drop scenario). It has a draw back that the accelerations during normal sinusoidal waves are higher, meaning that it has less comfort.

4 Simple gas spring

The simple gas spring is simply a cylinder filled with gas at bottom and rod side. The rod side is small compared to the bottom side, to make the influence of the rod side pressure on the force of the cylinder rather low. A simple overview

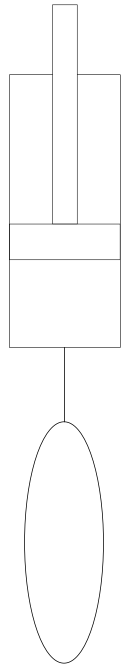

Figure 16: Graphical representation of the simple gas spring model.

of the system is shown in Figure 16.

The gas volume requires quite some more input values to make the simulation. The used inputs are shown in Table 4 The pressure at rod side p_{r0} is calculated using a isotherm process from the extended position to the x_0 position.

From the x_0 position the cylinder movement is considered adiabatic. The specific heat ratio κ changes with pressure and temperature, as shown in [2] and in Figure 17.

To calculate the cylinder force, first the stroke of the cylinder is determined:

$$x_{cyl} = x_2 - z_1 \tag{11}$$

To make sure that the thermodynamic calculation does not deliver Not a Number (NaN) answers, the maximum (0.1359[m]) and minimum (0[m]) stroke are forced. The to-

Table 4: Used quantities for the simple gas spring

Symbol	**quantity**	**unit**
D_{bore}	Bore diameter	$50[mm]$
D_{rod}	Rod diameter	$40[mm]$
V_{bottle}	Bottle bottom volume	$0.001[m^3]$
V_{rod_bottle}	Bottle rod volume	$0.001[m^3]$
x_0	Starting stroke	$0.0859[m]$
T_0	Ambient temperature	$20 + 273.15[K]$
$p_{r_prefill}$	Prefill pressure rod side retracted cylinder	$1\ 10^5[Pa]$

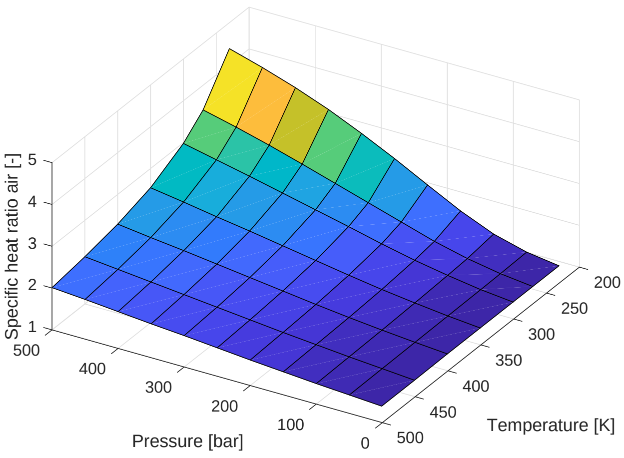

Figure 17: Change of the specific heat ratio of air with pressure and temperature.

tal volume is then calculated for rod and bottom side:

$$V_{bottom} = V_{bottle} + x_{cyl}\,\frac{\pi}{4}\,D_{bore}^2 = V_{bottle} + x_{cyl}\,A_{bore} \quad (12)$$

$$V_{rod} = V_{rodbottle} + (0.1359 - x_{cyl})\,\frac{\pi}{4}\,\left(D_{bore}^2 - D_{rod}^2\right) \quad (13)$$

$$= V_{rodbottle} + (0.1359 - x_{cyl})\,A_{rod} \quad (14)$$

The new volume is then used to calculate the new pressure and temperature using adiabatic calculation, using the adapted ideal gas law for pressure and temperature as shown in [2]. The cylinder force is then calculated as:

$$F_{cyl} = p_{bottom}\,A_{bore} - p_{rod}\,A_{rod} \quad (15)$$

The differential equation for the case that the suspension is not at the end of stroke is then:

$$\begin{bmatrix}\ddot{x}\\ \dot{x}\end{bmatrix} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}\frac{-m\,g + d\,(\dot{z}_1 - x_1) + F_{cyl}}{m}\\ x_1\end{bmatrix} \quad (16)$$

When the suspension hits the extended position ($x_2 - z_1 > 0.1359[m]$), the differential equation is changed to:

$$\begin{bmatrix}\ddot{x}\\ \dot{x}\end{bmatrix} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}\frac{\frac{-0.5*m\,(x_1 - \dot{z}_1)^2}{0.001} - m\,g + d\,(\dot{z}_1 - x_1)}{m}\\ x_1\end{bmatrix} \quad (17)$$

When the suspension hits the retracted position ($x_2 - z_1 < 0[m]$), the differential equation is changed to:

$$\begin{bmatrix}\ddot{x}\\ \dot{x}\end{bmatrix} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}\frac{\frac{0.5*m\,(x_1 - \dot{z}_1)^2}{0.001} + d\,(\dot{z}_1 - x_1) + F_{cyl}}{m}\\ x_1\end{bmatrix} \quad (18)$$

By using these differential equations, the simulations can be done as shown below.

4.1 Use it as gas spring

Here the pressure of the gas is set to the weight of the passenger. The pressure at bottom side at x_0 is thus dependent on the mass m:

$$p_{b0} = (m\,g + p_{r0}\,A_{rod})/A_{bore} \quad (19)$$

This results in a starting pressure p_{b0} of 4.6[bar].

With the gas pressure at bottom and rod side known, the cylinder force over the stroke can be determined, as shown in Figure 18. The stiffness from x_0 to extended stroke is on average 1712 [N/m]. The average stiffness from x_0 to extended stroke is 6440 [N/m]. On average (over the full stroke) the stiffness is 3451.8 [N/m], which is approximately equal to the linear spring model.

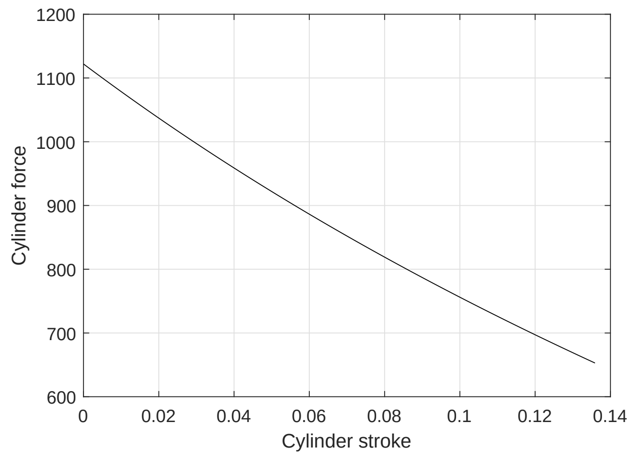

Figure 18: The cylinder force over the full stroke, including the 800[N] at 0.0859[m] stroke.

4.1.1 Results with wave motion

First the wave function is simulated. The result is shown in Figure 19. The temperature and cylinder force graph are shown in Figure 35. The peak acceleration is almost twice as high as for the linear and progressive spring, meaning that the gas spring is less comfortable than the other two.

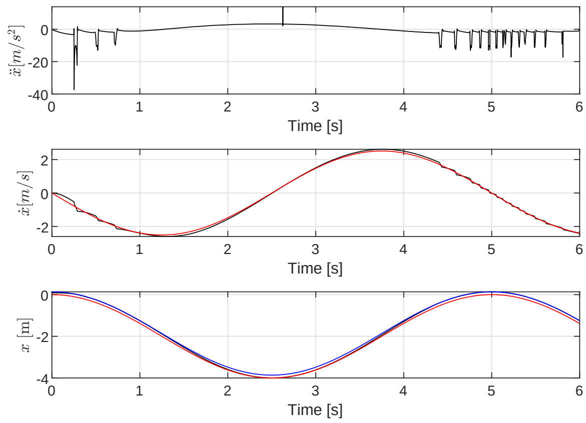

Figure 19: The acceleration, velocity and position for a wave function of the vessel.

4.1.2 Results for steep drop of the vessel

The results for a steep drop are shown in Figure 20. The temperature and cylinder force graph are shown in Figure 34. The results are comparable to the simple mechanical spring, and thereby better than the progressive mechanical spring.

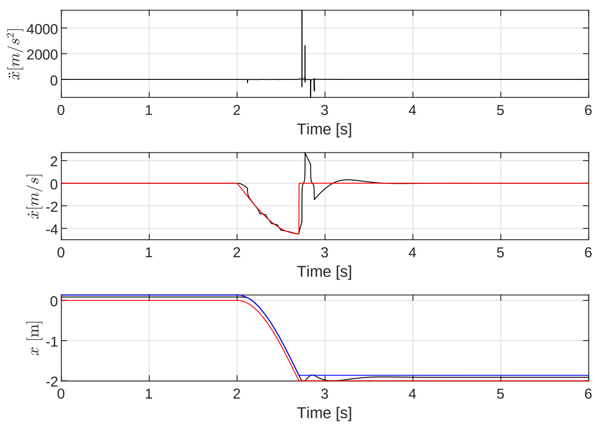

Figure 20: The acceleration, velocity and position for a steep drop of the vessel.

4.2 Set at one pressure independent of passenger weight

The pressure in the bottom side of the cylinder can be increased above the normal weight of the passenger, meaning that the suspension will normally be at the end of stroke (extended position). In this case the pressure is increased to 6.1[bar], which equals the 1200[N], which is again comparable with the linear spring.

The overal stiffness is 7579N/m over the full stroke. The cylinder force with respect to the stroke is shown in Figure 21.

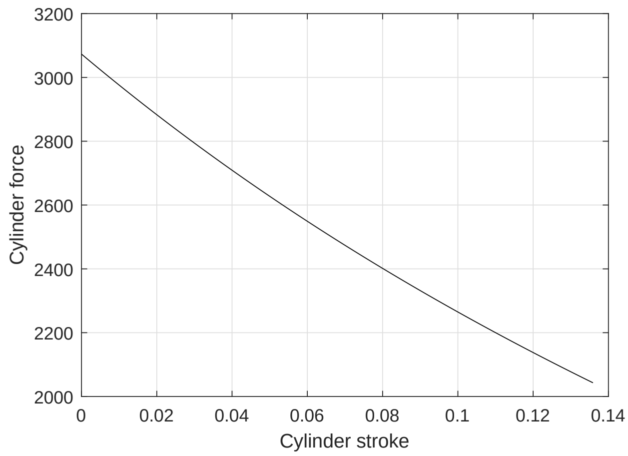

Figure 21: The cylinder force over the full stroke, including the 2750[N] at 0.0859[m] stroke.

4.2.1 Results with wave motion

The results for the wave motion are shown in Figure 22. The accelerations are approximately the same as for the initial simple gas system, and it is therefore a factor 2 larger than the linear and progressive mechanical spring with higher pretension.

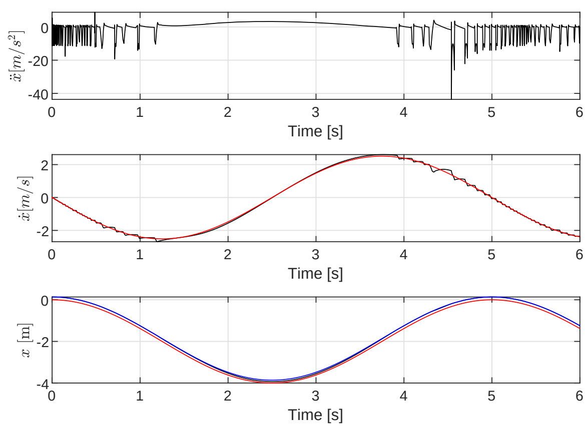

Figure 22: The acceleration, velocity and position for a wave function of the vessel with higher prefill pressure.

4.2.2 Results for steep drop of the vessel

The results for the higher prefill pressure for the simple gas system are shown in Figure 23. The accelerations are similar or higher than for the linear and progressive spring.

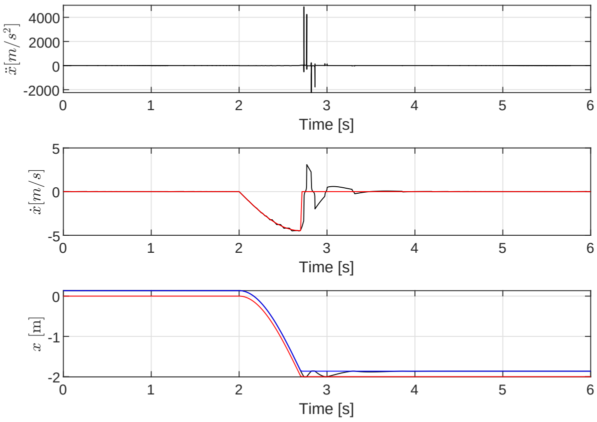

Figure 23: The acceleration, velocity and position for a steep drop of the vessel with higher prefill pressure.

4.3 Conclusion

The simple gas spring preset to the passengers weight has similar results for the steep drop as a mechanical spring. The sinusoidal wave shows that it is slightly less comfortable.

Increasing the pretension by increasing the pressure has no benefit as seen for the progressive mechanical spring.

5 Direction dependent gas spring

The gas spring only is not directly better than the mechanical spring. In order to limit the negative accelerations at the extended stroke, the gas volume is now separated: gas can quickly flow into the extra volume, meaning that the spring acts properly when the spring is compressed. During extending the spring , the gas is throttled before it enters the cylinder. This means that extending is done more slowly. This has the benefit that the suspension will not hit the extended stroke with the same speed as before, which should result in lower negative accelerations.

This is immediately one of the drawbacks: it should limit the negative accelerations, but especially for the steep drop, the largest accelerations are in the positive direction due to hitting the retracted stroke position. It has the disadvantage of slower extending, meaning that the suspension is longer retracted and is thus also for a longer period of time not capable of dampening new larger shocks.

5.1 Calculation method

To make a clear distinction between the in- and outgoing stroke, the two are shown separately here.

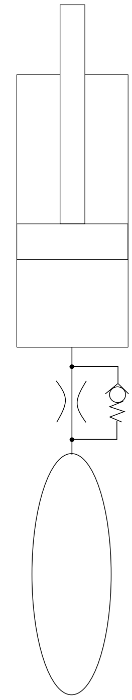

Figure 24: Graphical representation of the adapted gas spring model.

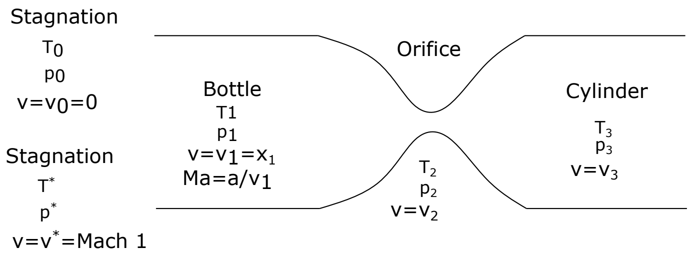

Figure 25: Model of the orifice.

5.1.1 Ingoing stroke

The ingoing stroke is defined when the speed of the cylinder piston is smaller or equal to zero:

$$x_1 - \dot{z}_1 \leq 0 \tag{20}$$

The gas pressure and temperature calculation is similar as for the simple gas spring. Just to make sure that the simulation runs on the correct way, the following global values are reset in this situation to their appropriate values:

$$M_{1_n} = \frac{p_1 \, V_{bottle}}{Z_{value}(p,T) \, R_{specific} \, T_1} \tag{21}$$

$$M_{3_n} = \frac{p_3 \, A_{bore} \, (x_1 - z_1)}{Z_{value}(p,T) \, R_{specific} \, T_3} \tag{22}$$

$$p_3 = p_1 \tag{23}$$

During the ingoing stroke, the equations of motion are the same as seen in Section 4. The equations of motion will be repeated here for clarity and comparison with the outgoing stroke. When the suspension is not at the end of stroke, the differential equation is:

$$\begin{bmatrix} \ddot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{-m\,g + d\,(\dot{z}_1 - x_1) + F_{cyl}}{m} \\ x_1 \end{bmatrix} \tag{24}$$

When the suspension hits the extended position ($x_2 - z_1 > 0.1359[m]$), the differential equation is changed to:

$$\begin{bmatrix} \ddot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{\frac{-0.5*m\,(x_1 - \dot{z}_1)^2}{0.001} - m\,g + d\,(\dot{z}_1 - x_1)}{m} \\ x_1 \end{bmatrix} \tag{25}$$

When the suspension hits the retracted position ($x_2 - z_1 < 0[m]$), the differential equation is changed to:

$$\begin{bmatrix} \ddot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{\frac{0.5*m\,(x_1 - \dot{z}_1)^2}{0.001} + d\,(\dot{z}_1 - x_1) + F_{cyl}}{m} \\ x_1 \end{bmatrix} \tag{26}$$

5.1.2 Outgoing stroke

For the outgoing stroke ($x_1 - \dot{z}_1 > 0$), the bottom volume of the cylinder and the extra volume V_{bottle} are coupled with the orifice. This means that the two volumes do not necessarily have the same pressure.

The laws used up till now are all based on a closed and specific volume without mass transfer over the system boundaries. For the outgoing stroke, the system needs to be divided into two volumes and there is a mass flow between the two volumes. Therefore, it is required to change the laws used for the simulation.

In order to compensate for the lack of rules, the first law of thermodynamics is used: The law of conservation of energy. In this case, where atomic reactions are not considered within the system, it also means the conservation of mass.

The conservation of mass means that in the total volume of cylinder and V_{bottle} the mass does not increase or decrease, although the mass can flow from one to the other.

From the starting positions p_0, T_0 and V_0 at the bottom side the total mass can be determined, as shown in [2]:

$$M(p_0, V_0, T_0) = \frac{p_0\,V_0}{Z_{value}(p,t)\,R_n\,T_0} \tag{27}$$

Table 5: Explanation of the quantities

Symbol	quantity	unit
A	Area before orifice	$[m^2]$
A^*	Critical area (sonic flow)	$[m^2]$
$A_{orifice}$	Area of the orifice	$[m^2]$
$D_{orifice}$	Diameter of orifice	$1\,10^{-3}[m]$
k	Specific heat ratio	$[-]$
Ma	Mach number	$[-]$
p	Pressure before orifice	$[Pa]$
p_0	Stagnation pressure	$[Pa]$
$R_{specific}$	Specific gas constant	$268.9[\frac{J}{k\,kg}]$
T	Temperature before orifice	$[K]$
T_0	Stagnation temperature	$[K]$
ρ	Density of the gas	$[kg/m^3]$
ρ_0	Stagnation density	$[kg/m^3]$

For each step, the mass flow over the orifice needs to be calculated. As this is compressible flow, the critical orifice area needs to be determined in order to determine whether the flow is considered choked or subsonic flow ([3] page 638). To calculate the critical area, the mach number at point 1 ([3] page 629 and 632) needs to be determined at each point:

$$Ma = \frac{v}{\sqrt{k\,R_{specific}\,T_1}} = \frac{x_1 - \dot{z}_1}{\sqrt{k\,R_{specific}\,T_1}} \tag{28}$$

$$\frac{A_1}{A^*} = \frac{1}{Ma}\left(\frac{1 + \frac{1}{2}\,(k-1)\,Ma^2}{\frac{1}{2}\,(k+1)}\right)^{\frac{1}{2}\,(k+1)\,(k-1)} \tag{29}$$

$$A^* = \frac{A_1\,Ma}{\left(\frac{1+\frac{1}{2}\,(k-1)\,Ma^2}{\frac{1}{2}\,(k+1)}\right)^{\frac{1}{2}\,(k+1)\,(k-1)}} \tag{30}$$

In order to calculate the mass flow, the **stagnation point** parameters, depicted with a 0, need to be determined. The stagnation point is the point at which the fluid velocity becomes zero. These can be calculated:

$$T_0 = \left(1 + \frac{k-1}{2}\,Ma^2\right)\,T \tag{31}$$

$$p_0 = \left(1 + \frac{1}{2}\,(k-1)\,Ma^2\right)^{\frac{k}{k-1}}\,p \tag{32}$$

$$\rho_0 = \left(1 + \frac{1}{2}\,(k-1)\,Ma^2\right)^{\frac{1}{k-1}}\,\rho \tag{33}$$

With a orifice area larger than the critical area ($A_{orifice} > A^*$), the Mach number is smaller than one (< 1) and the flow is subsonic, and can be calculated ([3] page 640):

$$\dot{m} = \frac{A\,p_0}{\sqrt{R_{specific}\,T_0}}\sqrt{\frac{2\,k}{k-1}\left(\frac{p}{p_0}\right)^{2\,k}\left(1 - \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}\right)} \tag{34}$$

With a orifice area equal or smaller than the critical area ($A_{orifice} \leq A^*$), the Mach number equal to one (=1) and the mass flow is at the maximum and becomes ([3] page 639):

$$\dot{m} = \dot{m}_{max} = \frac{0.6847\,p_0\,A^*}{\sqrt{R_{specific}\,T_0}} \tag{35}$$

This means that within the simulation time step dt the mass entering the cylinder is:

$$dM = \dot{m}\,dt \tag{36}$$

The change of mass requires thus the dt, which is the reason that the standard variable integrators cannot be used. Therefore, the constant time step integrator according the fourth order Ranga-Kutta method is used for this system.

Now that the mass flow is known, the pressure in the cylinder needs to be determined. To start, the mass of the gas at the start of the simulation is determined based on the compressibility factor theory [2]:

$$M_{start} = \frac{p_{start}\,V_{start}}{Z_{value}(p,T)\,R_{specific}\,T_{start}} \tag{37}$$

To calculate the resulting pressure and temperature in the bottom bottle, the density is determined first using the mass of the previous time step (n-1)[1]:

$$\rho_1 = \frac{M_{1_{n-1}}}{V_{bottle}} \tag{38}$$

$$\rho_3 = \frac{M_{3_{n-1}}}{A_{bore}\,(x_2 - z_1)} \tag{39}$$

Now that the estimation of the density is done, the estimation of the volume change due to the mass flow can be determined:

$$dV_1 = \frac{dM}{\rho_1} \tag{40}$$

$$dV_3 = \frac{dM}{\rho_3} \tag{41}$$

By using the change in volume, the new temperature can be determined, which is again used to determine the new pressure.

This means that the temperature and pressure at the new time step (n) are:

$$T_{1_n} = T_{1_{n-1}} \left(\frac{V_{1_{n-1}}}{V_{1_{n-1}} + dV} \right)^{k-1} \tag{42}$$

$$T_{3_n} = T_{3_{n-1}} \left(\frac{V_{3_{n-1}}}{V_{3_{n-1}} - dV + A_bore\,(x_1 - \dot{z}_1\,dt)} \right)^{k-1} \tag{43}$$

$$M_{1_n} = M_{1_{n-1}} - dM \tag{44}$$

$$M_{3_n} = M_{3_{n-1}} + dM \tag{45}$$

$$p_{1_n} = \frac{Z_{value}(p,T)\,M_{1_n}\,R_{specific}\,T_{1_n}}{V_{bottle}} \tag{46}$$

$$p_{3_n} = \frac{Z_{value}(p,T)\,M_{3_n}\,R_{specific}\,T_{3_n}}{A_{bore}\,(x_1 - z_1 + (x_1 - \dot{z}_1\,dt)} \tag{47}$$

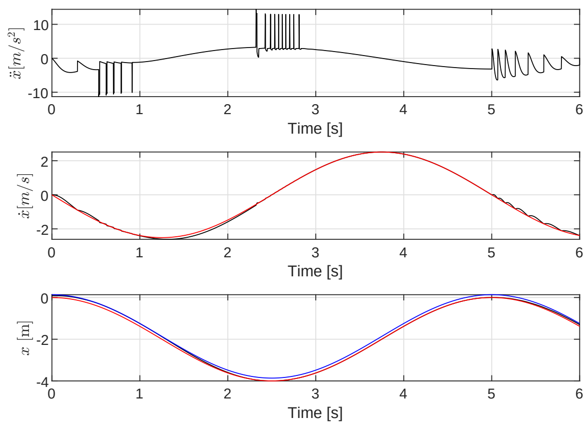

Figure 26: The acceleration, velocity and position for a wave function of the vessel.

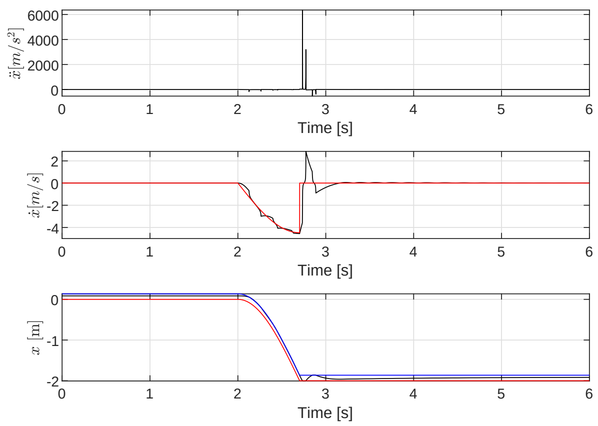

Figure 27: The acceleration, velocity and position for a steep drop of the vessel.

The rod pressure is determined similarly as for the simple gas system.

The cylinder force can now be calculated using:

$$F_{cyl} = p_3\,A_{bore} - p_{rod}\,A_{rod} \tag{48}$$

The equations of motion are the same as seen in Section 4.

$$\begin{bmatrix} \ddot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{-m\,g + d\,(\dot{z}_1 - x_1) + F_{cyl}}{m} \\ x_1 \end{bmatrix} \tag{49}$$

When the suspension hits the extended position ($x_2 - z_1 > 0.1359[m]$), the differential equation is changed to:

$$\begin{bmatrix} \ddot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{\frac{-0.5*m\,(x_1 - \dot{z}_1)^2}{0.001} - m\,g + d\,(\dot{z}_1 - x_1)}{m} \\ x_1 \end{bmatrix} \tag{50}$$

When the suspension hits the retracted position ($x_2 - z_1 < 0[m]$), the differential equation is changed to:

$$\begin{bmatrix} \ddot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{\frac{0.5*m\,(x_1 - \dot{z}_1)^2}{0.001} + d\,(\dot{z}_1 - x_1) + F_{cyl}}{m} \\ x_1 \end{bmatrix} \tag{51}$$

5.2 Results for spring adjusted to weight passenger

5.2.1 Wave motion

First the wave motion is simulated. The results are shown in Figure 26. The negative accellerations are lower, which corresponds with a slower speed at which the chair hits the end of stroke position. This means that the air flow for extending is throttled, slowing down the motion. The gas temperature and cylinder force are shown in Figure 38.

5.2.2 Steep drop

The result of the simulation for the steep drop are shown in Figure 27. The positive accelerations are still quite high. but the negative accelerations are again lower. Unfortunately, the positive accelerations are larger in this case, as

[1] Using the mass of the previous time step results in a small offset, but as the time steps are small and thus the mass steps are even smaller, the error will be very small.

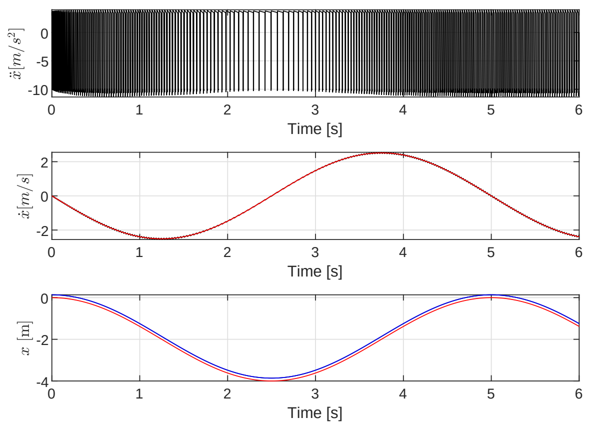

Figure 28: The acceleration, velocity and position for a wave function of the vessel with higher prefill pressure.

the largest impact is at the retracted stroke (and the impact force slowing down the mass is thus in upward direction). This means that the benefit for the person in the chair is small.

5.3 Results for prefill pressure independent of passenger weight

Similar as done previously, the pressure in the cylinder can be increased to 6,1bar. This means that the chair will start fully extended.

5.3.1 Wave motion

The results are shown in Figure 28. The accelerations are smaller compared to Figure 22, meaning it is a more comfortable seat.

5.3.2 Steep drop

The results for the steep drop with higher gas spring pressure is shown in Figure 29. It shows almost no negative accelerations, which is an improvement. The positive accelerations are however still the same, which were governing.

5.4 Remove the check valve

From the results displayed above, one might think that removing the check valve, and thus only installing an orifice between the cylinder and the bottle, would work better, as the orifice lowers the accelerations.

To show that this is not directly a solution, a simulation is done with only the orifice for a steep drop and the pressure adjusted to the person's weight. This simulation is shown in Figure 30. It is shown that the accelerations are now still quite high, but slightly lower, for the positive accelerations (hit end of stroke at bottom side). The negative accelerations are however much worse, meaning that

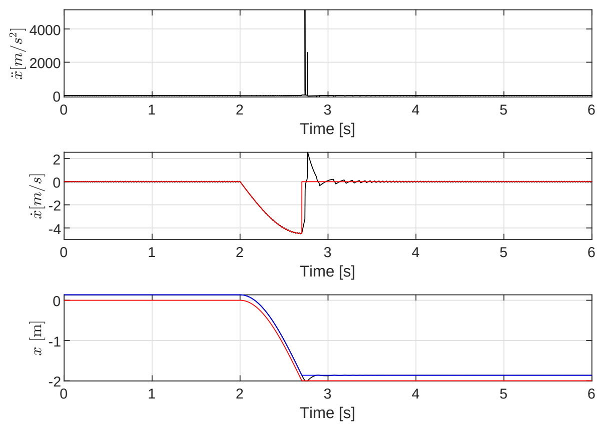

Figure 29: The acceleration, velocity and position for a steep drop of the vessel with higher prefill pressure.

the total amount of accelerations are higher and this design is thus less favourable for the passenger.

5.5 Conclusion

The conclusion of this section is that the throttle check valve is beneficial for the negative accelerations compared to the simple gas spring. However, the largest accelerations are in the positive direction and these are still the same as the simple gas spring.

6 Active constant pressure system

The last idea to be modeled and simulated, is an active constant pressure system. This system uses an active system to add gas when the pressure is too low, and removes gas when the pressure is too high. Goal is to support the person on the chair as properly as possible.

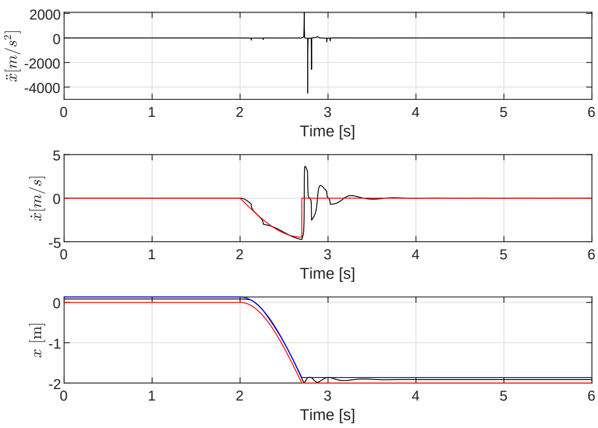

Figure 30: The acceleration, velocity and position for a steep drop of the vessel.

However, a small difference between added gas and relieved gas is required, to prevent a constant energy usage, when the gas is added from a compressor while it is immediately relieved through the relief valve. Therefore, an estimate of 1bar pressure difference is assumed to be between the compressor and the relief valve. An overview of the system is shown in Figure 31.

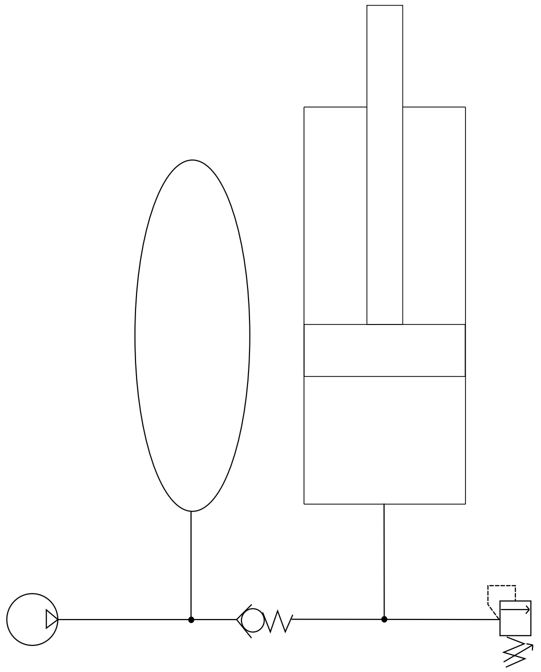

Figure 31: Graphical representation of the adapted gas spring model.

The calculation looks a lot like the calculation done for the simple gas spring of chapter 4, with the exception that the minimum pressure is kept equal to the weight of the passenger, while the maximum pressure is 1 bar above that setting[2].

To calculate the cylinder force, first the stroke of the cylinder is determined:

$$x_{cyl} = x_2 - z_1 \tag{52}$$

To make sure that the thermodynamic calculation does not deliver Not a Number (NaN) answers, the maximum (0.1359[m]) and minimum (0[m]) stroke are forced. The total volume is then calculated for rod and bottom side:

$$V_{bottom} = V_{bottle} + x_{cyl}\,\frac{\pi}{4}\,D_{bore}^2 = V_{bottle} + x_{cyl}\,A_{bore} \tag{53}$$

$$V_{rod} = V_{rodbottle} + (0.1359 - x_{cyl})\,\frac{\pi}{4}\,\left(D_{bore}^2 - D_{rod}^2\right) \tag{54}$$

$$= V_{rodbottle} + (0.1359 - x_{cyl})\;A_{rod} \tag{55}$$

The new volume is then used to calculate the new pressure and temperature using adiabatic calculation, using the adapted ideal gas law for pressure and temperature as shown in [2]. The temperature of the gas is in that case set to the starting temperature T_0. The cylinder force is then calculated as:

$$F_{cyl} = p_{bottom}\,A_{bore} - p_{rod}\,A_{rod} \tag{56}$$

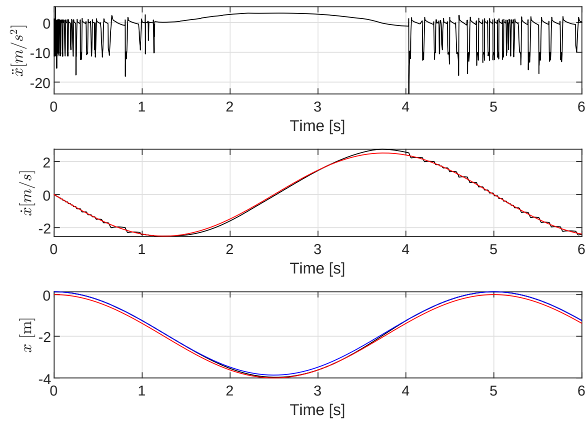

Figure 32: The acceleration, velocity and position for a wave function of the vessel with higher prefill pressure.

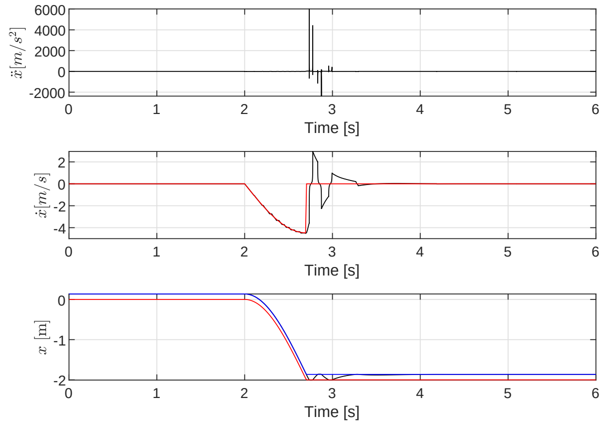

Figure 33: The acceleration, velocity and position for a steep drop of the vessel.

The differential equation for the case that the suspension is not at the end of stroke is then:

$$\begin{bmatrix}\ddot{x}\\ \dot{x}\end{bmatrix} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}\frac{-m\,g + d\,(\dot{z}_1 - x_1) + F_{cyl}}{m}\\ x_1\end{bmatrix} \tag{57}$$

When the suspension hits the extended position ($x_2 - z_1 > 0.1359[m]$), the differential equation is changed to:

$$\begin{bmatrix}\ddot{x}\\ \dot{x}\end{bmatrix} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}\frac{\frac{-0.5*m\,(x_1 - \dot{z}_1)^2}{0.001} - m\,g + d\,(\dot{z}_1 - x_1)}{m}\\ x_1\end{bmatrix} \tag{58}$$

[2] 1bar is chosen, as the actual pressure is about 4.6bar, and for larger differences the relief valve would not do much good.

When the suspension hits the retracted position ($x_2 - z_1 < 0[m]$), the differential equation is changed to:

$$\begin{bmatrix} \ddot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{\frac{0.5 * m \, (x_1 - \dot{z}_1)^2}{0.001} + d \, (\dot{z}_1 - x_1) + F_{cyl}}{m} \\ x_1 \end{bmatrix} \tag{59}$$

6.1 Results for spring adjusted to weight passenger

The pressure of the gas is set to the weight of the passenger. The pressure at bottom side at x_0 is thus dependent on the mass m:

$$p_{b0} = (m \, g + p_{r0} \, A_{rod}) / A_{bore} \tag{60}$$

This results in a starting pressure p_{b0} of 4.6[bar].

6.1.1 Wave motion

First the wave motion is simulated. The results are shown in Figure 32. The accelerations are lower than for a simple gas spring, meaning that the seat provides a more comfortable ride of the passenger. The gas temperature and cylinder force are shown in Figure 43.

6.1.2 Steep drop

The result of the simulation for the steep drop are shown in Figure 33. The positive as well as negative accelerations are much higher than for other systems. This has to do with the fact that the chair cannot slow down the motion by building a higher pressure or force, which is possible for the other designs. This means that although the ride is smoother as long as the end of stroke position is not reached, it will mean that as soon as it hits the end of stroke position, it will do greater damage. And as the design lacks the possibility to slow down the motion, it will hit the end of stroke position for smaller waves than the other designs.

7 Conclusion

The active system is not the system to choose, as it lacks the ability to slow down movement, as it cannot develop a force to slow down the movement. The other systems have a trade-off between comfort (sinusoidal movement) and accelerations at maximum operational conditions (steep drop). The progressive mechanical spring with extra pre-tension seems favorable for large steep drops, while the direction dependent gas spring adjusted to the passenger's weight has more comfort for the passenger.

Important to all simulations is that the large accelerations are due to the end-of-stroke of the suspension. This means that once the stroke can be enlarged, it has significant benefits, as it requires larger movements to reach the end-of-stroke position. This larger stroke is however not simulated in this article.

A higher spring stiffness, as for the progressive mechanical spring, has benefits, as it limits the speed at which the end-of-stroke position is reached. The better the passenger is slowed down, the lower the impact and thus the lower the accelerations and thus forces on the passenger. For the gas springs, it might also be possible to reduce the gas volume to create this stiffer spring.

All in all, the design of such a chair should represent the maximum operational conditions. Enlarging the stroke cannot be done indefinitely, but it also comes with higher costs and with a larger space claim. Therefore, the maximum operational conditions for each chair and vessel needs to be determined to have a cost-effective solution.

References

[1] J.G. Gruijters. Basics of linear control theory for mechanical engineering. *Not published*, 2018.

[2] J.G. Gruijters. The compressability of gas. *Not published*, 2018.

[3] Frank M. White. *Fluid Mechanics*. McGraw Hill, 2011.

Appendices

A Simple Gas System, adapted to weight

A.1 Wave motion

The cylinder force and temperature of the gas volumes of the simple gas system during the wave motion are shown in Figure 34.

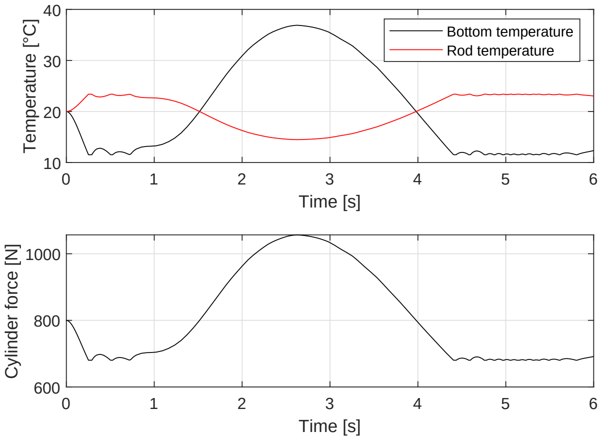

Figure 34: The temperature and cylinder force for a wave function of the vessel.

A.2 Steep drop

The cylinder force and temperature of the gas volumes of the simple gas system during the steep drop are shown in Figure 35.

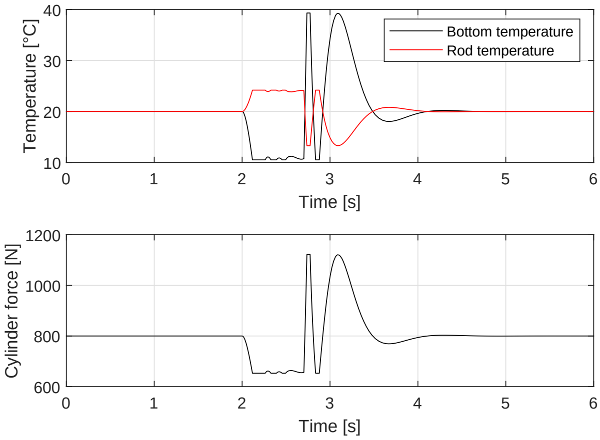

Figure 35: The temperature and cylinder force for a steep drop of the vessel.

B Simple Gas System, independent of passenger weight

B.1 Wave motion

The cylinder force and temperature of the gas volumes of the simple gas system during the wave motion are shown in Figure 36.

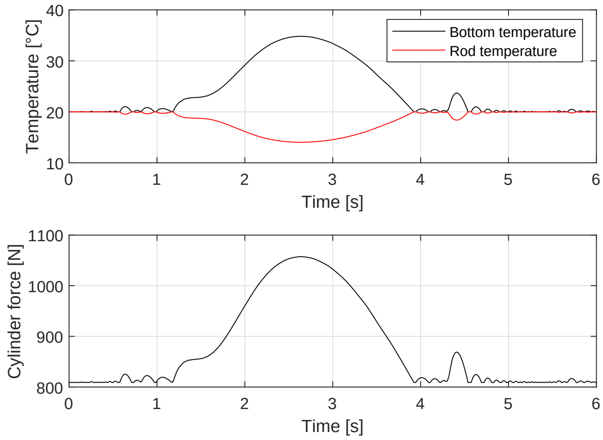

Figure 36: The temperature and cylinder force for a wave function of the vessel.

B.2 Steep drop

The cylinder force and temperature of the gas volumes of the simple gas system during the steep drop are shown in Figure 37.

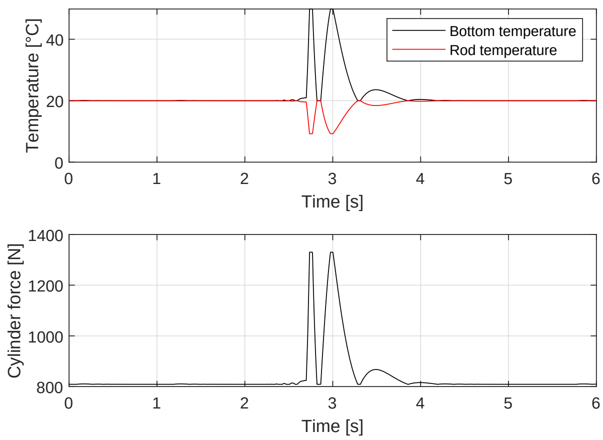

Figure 37: The temperature and cylinder force for a steep drop of the vessel.

C Direction dependent gas system, adapted to weight

C.1 Wave motion

The cylinder force and temperature of the gas volumes of the simple gas system during the wave motion are shown in Figure 38.

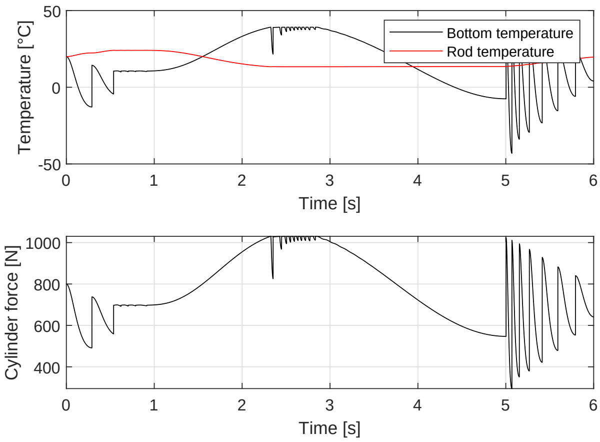

Figure 38: The temperature and cylinder force for a wave function of the vessel.

C.2 Steep drop

The cylinder force and temperature of the gas volumes of the simple gas system during the steep drop are shown in Figure 39.

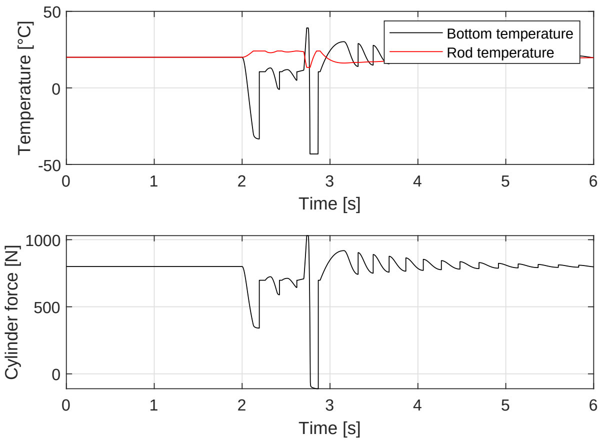

Figure 39: The temperature and cylinder force for a steep drop of the vessel.

C.3 Steep drop, removed check valve

In Figure 40 the cylinder force and temperature of the gas are shown for the steep drop simulation with only an orifice, thus where the check valve is removed.

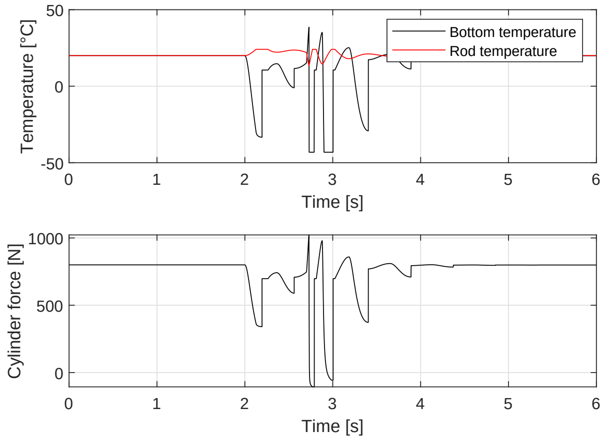

Figure 40: The temperature and cylinder force for a steep drop of the vessel.

D Direction dependent gas system, independent of passenger weight

D.1 Wave motion

The cylinder force and temperature of the gas volumes of the simple gas system during the wave motion are shown in Figure 41.

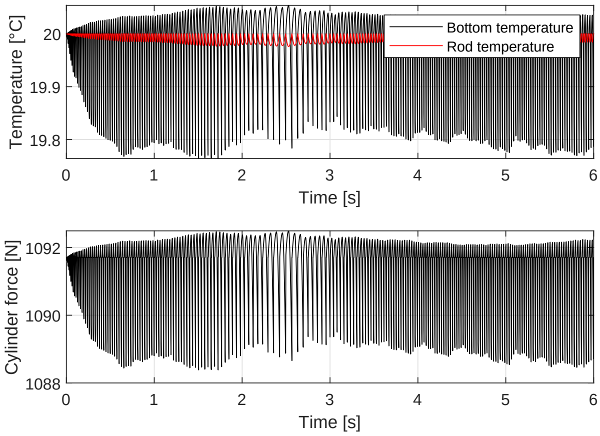

Figure 41: The temperature and cylinder force for a wave function of the vessel.

D.2 Steep drop

The cylinder force and temperature of the gas volumes of the simple gas system during the steep drop are shown in Figure 42.

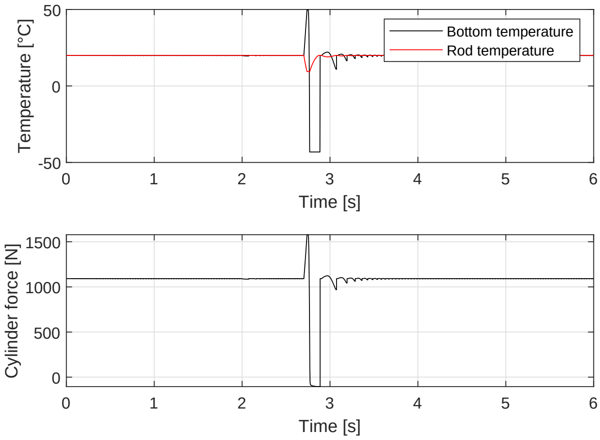

Figure 42: The temperature and cylinder force for a steep drop of the vessel.

E Active constant pressure system

E.1 Wave motion

The cylinder force and temperature of the gas volumes of the simple gas system during the wave motion are shown in Figure 43.

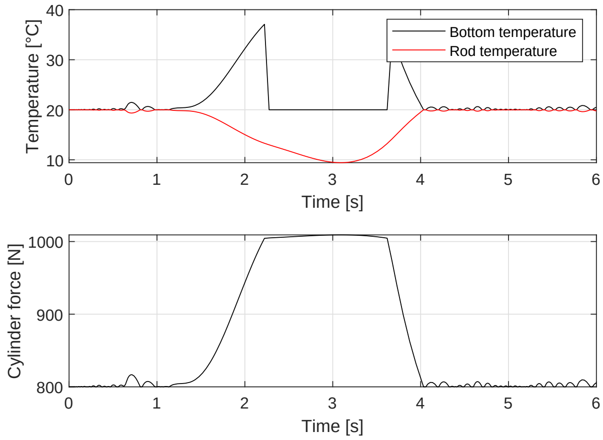

Figure 43: The temperature and cylinder force for a wave function of the vessel.

E.2 Steep drop

The cylinder force and temperature of the gas volumes of the simple gas system during the steep drop are shown in Figure 44.

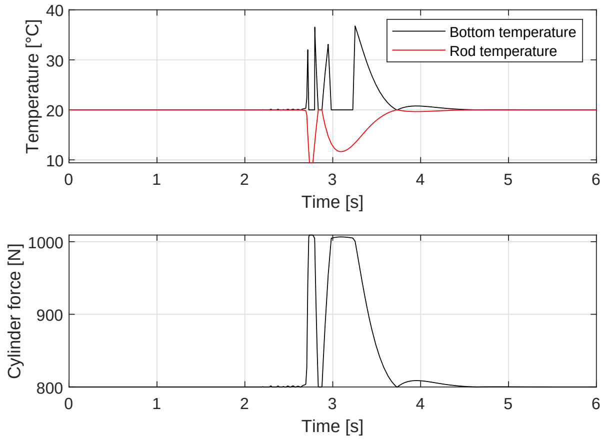

Figure 44: The temperature and cylinder force for a steep drop of the vessel.