

Magnetism and Electric Systems

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Abstract

This article shows the theory required to study the frequency drives and the control of these drives. This overview is made while studying electrical drives [6] and should be seen as notes taken during this study.

This article shows the theory required to further study the frequency drives. The laws of Kirchhoff are already shown in Section 2 of [5], so that will not be repeated here. The magnetic circuits will be further explored.

1 Magnetic circuits

The theory of magnetic circuits are required to learn more about transformers, which are used in variable frequency drives, AC motors and generators.

1.1 Ampere's Law - Field intensity

Every conductor that has a current i flowing through it, produces a magnetic field. This is shown in Figure 1. The magnetic field is drawn as a dashed line and the direction of the field is shown by the arrows. The field intensity H produced by current-carrying conductors can be obtained by Ampere's law:

$$\oint H dl = \sum i \quad (1)$$

where the \oint represents the integral over a contour. H is the magnetic field intensity in the direction of dl along the closed path. Alternatively, the field intensity H and differential length dl can be expressed as a vector, meaning that the left hand side Equation 1 becomes the dot product.

Just to show how simple the Ampere's law is, a small example is shown below.

In Figure 2 a toroid is shown with a wire (conductor) wrapped around it. Due to symmetry the magnetic field intensity H_m along a circular contour l_m in the middle of the toroid is constant. The mean radius r_m of the contour

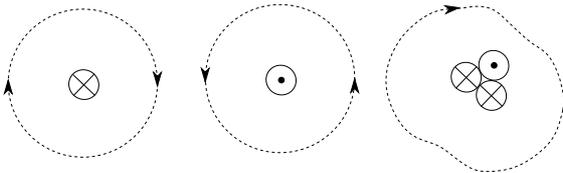


Figure 1: Graphical representation of the magnetic field as per Ampere's law.

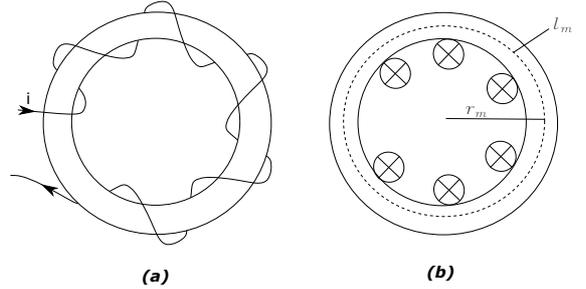


Figure 2: Example for Ampere's law.

is calculated by:

$$r_m = \frac{1}{2} \frac{OD + ID}{2} \quad (2)$$

The length of the contour is a circle and therefore:

$$l_m = 2 \pi r_m \quad (3)$$

The contour l_m encloses the current n times, which leads to:

$$H_m = \frac{n i}{l_m} \quad (4)$$

Note that the magnetic field H_m has unit $[A/m]$. The product of $n i$ is also referred to as 'Ampere-turns'. The current can either be DC or AC. When the current is time-varying (AC), the relationship as shown in Equation 4 is valid on an instantaneous basis, so $H_m(t)$ is dependent on $i(t)$ and constant factor n/l_m .

To calculate the magnetic field strength of a coil (inside the coil), the following formula can be used:

$$H = \frac{i n}{L} \quad (5)$$

Where i ($[A]$) is the current, n ($[-]$) the amount of turns of the coil and L ($[m]$) is the length of the coil.

To calculate the magnetic field strength of a straight conductor, the following can be used:

$$H = \frac{i}{2 \pi r} \quad (6)$$

Where i ($[A]$) is the current and r ($[m]$) is the radius if the at which the field is measured.

1.2 Flux and flux density

Above the magnetic field density is shown using Ampere's law. Flux is defined as a force field in a specific medium, or in other words, flow of energy through a surface (i.e. the magnetic field lines drawn around a magnet). The magnetic flux thus describes the total magnetic field passing through a given area. This quantity is useful to describe effects of magnetic force on something occupying that certain area (e.g. a core).

The magnetic flux density B (in units Tesla $[T]$ or Weber per squared meter $[Wb/m^2]$), which is thus the density of energy flowing through the surface area, depends on the permeability μ of the material occupying the surface on which the field H (as seen above) is acting:

$$B = \mu H \quad (7)$$

The permeability is the measure for the degree of magnetization of a material in response to a magnetic field. The unit is Henries per meter (H/m). This leads to the realisation that $[T] = [Wb/m^2] = [H/m][A/m] = [(H A)/m^2]$ and thus $[Wb] = [H A]$.

1.2.1 Ferromagnetic materials

When looking at Equation 4 from the example above and Equation 7, the following is obtained:

$$B_m = \mu_m \frac{n i}{l_m} \quad (8)$$

This means that for materials with a higher permeability will need less Ampere-turns (and thus a smaller current for a given amount of turns) to produce the same flux density.

Imagine that the toroid in Figure 2 is made of ferromagnetic material, e.g. silicon steel (often called electrical steel due to the small hysteresis area and thus low power losses and high permeability). When varying the current through the wire slowly with a sinusoidal manner, the flux will behave with some hysteresis as shown in Figure 3 (a). The hysteresis results in dissipation of energy, thus a power loss, and is therefore called hysteresis loss.

Figure 3 (b) shows a line where all peaks of the graphs of (a) are joint by a line.

Increasing the amplitude of the current will result in a bigger hysteresis loop. But above a certain current, the loop

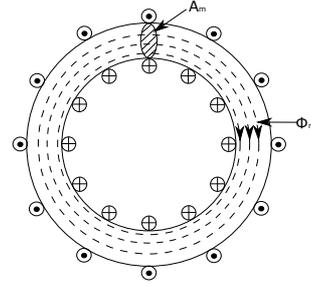


Figure 4: Toroid with flux lines drawn in it

shows the non-linear effect, which means that the line bends to a much lower gradient. Here, much more Ampere-turns are required to increase the flux density. This is called the saturation zone and is often not used in designs, as it is not very efficient to increase the magnetic field. The saturation is often found between 1.6 and 1.8[T]. Using only the linear part leads to the linearized equation:

$$B_m = \mu_m H_m \quad (9)$$

where μ_m is the permeability of the ferromagnetic toroid. The permeability is often expressed in terms of a relative permeability μ_r , relative to the permeability of air:

$$\mu_r = \frac{\mu_m}{\mu_0} \quad (10)$$

The permeability of air $\mu_0 = 4 \pi 10^{-7} \frac{H}{m}$. The permeability of ferromagnetic materials can be several to even tens of thousands times larger compared to air.

1.2.2 Flux

Magnetic flux lines form closed paths in the toroid, as shown in Figure 4. The flux Φ (in Weber $[Wb]$) is calculated by selecting an area A_m in a **plane perpendicular to the flux lines**. The flux then is ¹:

$$\Phi_m = B_m A_m \quad (11)$$

It is reasonable to assume a uniform H_m and thus a uniform B_m . Combining Equation 8 and 11 then gives:

$$\Phi_m = A_m \mu_m \frac{n i}{l_m} = \frac{n i}{\frac{l_m}{A_m \mu_m}} = \frac{n i}{\mathfrak{R}_m} \quad (12)$$

Where \mathfrak{R}_m (unit $[A/Wb]$) is the reluctance of the magnetic core, which describes the 'unwillingness' of the material to let the flux pass. It is the magnetic analogy to the electrical resistance for electrical circuits, similar as Ohm's law ($I = V/R$ for DC system see [5] for the general term impedance). This also shows that $[A/Wb] = [1/H]$ and thus $[H] = [Wb/A]$.

¹Note here that the equation below is only true when the flux lines are perpendicular to the area. If not, this equation must be multiplied with $\cos(\theta)$, where θ is the angle of the flux lines with with the perpendicular line.

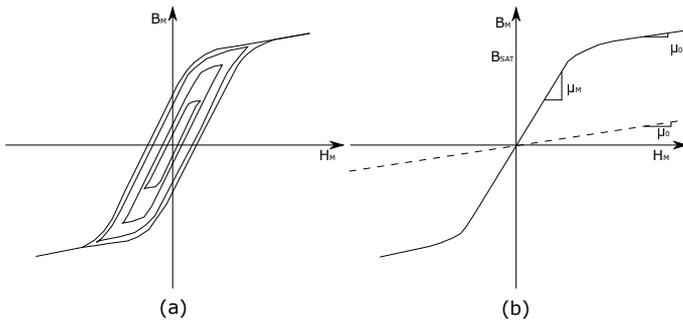


Figure 3: B-H characteristic of ferromagnetic materials

If all turns of a coil are linked by the same flux Φ , such as in figure 4, then the coil has flux linkage λ^2 :

$$\lambda = n \Phi \quad (13)$$

1.2.3 Magnetic structures with air gaps

Up till now, the closed path went through one material only. Now there is an air gap in the closed path, see Figure 5. To start with Ampere's law (Equation 1):

$$H_m l_m + H_g l_g = n i \quad (14)$$

The flux density is then inserted in the equation:

$$B_m = \mu_m H_m \quad (15)$$

$$B_g = \mu_0 H_g \quad (16)$$

$$\frac{B_m}{\mu_m} l_m + \frac{B_g}{\mu_0} l_g = n i \quad (17)$$

The flux lines form closed paths, and thus is the flux density through the material as well as the air gap the same:

$$\Phi = A_m B_m = A_g B_g \quad (18)$$

This leads to:

$$\Phi \left(\frac{l_m}{A_m \mu_m} + \frac{l_g}{A_g \mu_0} \right) = n i \quad (19)$$

The equations above assume that the flux lines are straight, however in air the lines bulge a bit, as already shown in Figure 5. This bulging is called the *fringing effect*. Due to the fringing effect, the air gap area is now calculated using:

$$A_g = (W + l_g) (d + l_g) \quad (20)$$

As $\mathfrak{R}_m = \frac{l_m}{A_m \mu_m}$, $\mathfrak{R}_g = \frac{l_g}{A_g \mu_0}$, $\mathfrak{R} = \mathfrak{R}_m + \mathfrak{R}_g$ and the magneto-motive force (MMF) $F = n i$:

$$\Phi = \frac{F}{\mathfrak{R}} \quad (21)$$

As both F and \mathfrak{R} can be calculated, the flux can be calculated. Afterward, the flux density B_m and B_g can be calculated using Equation 18, which makes all parts solvable. Note that the air gap reluctance \mathfrak{R}_g is normally dominating, meaning that for (rough) estimations the core reluctance \mathfrak{R}_m can be neglected in Equation 21.

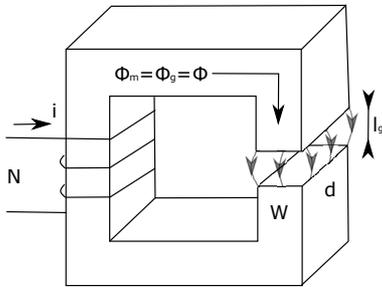


Figure 5: Magnetic structure with air gap

²Note here that the equation below is only true when the flux lines are perpendicular to the area. If not, this equation must be multiplied with $\cos(\theta)$, where θ is the angle of the flux lines with with the perpendicular line.

1.3 Inductance

1.3.1 Relations of Inductance with flux linkage

When looking at Figure 4, the flux linkage λ (Equation 13) and the current are linked together by the inductance L_m :

$$\lambda_m = L_m i \quad (22)$$

Where the inductance L_m (in H or Henry) is constant if the core material is in its linear operating region.

By combining Amperes law (Equation 4), flux density (Equation 7), flux (Equation 11) and flux linkage (Equation 13) the inductance is determined:

$$L_m = \frac{n}{l_m} \mu_m A_m n \quad (23)$$

$$L_m = \frac{n^2}{\left(\frac{l_m}{\mu_m A_m} \right)} = \frac{n^2}{\mathfrak{R}_m} \quad (24)$$

This shows that, as long as the operation is in the linear range, the inductance is only influenced by the property of the magnetic circuit (i.e. material, geometry and number of turns) with slope μ_m on a B - H diagram.

1.3.2 Magnetic energy storage in inductors

Now that the main players for inductors are explained, it is time to look at the energy storage in the inductor. This is similar to the work required to produce a current through the inductor.

To know more about the inductance, and to understand the energy storage better, it is good to have an example. Imagine switching the light in a room on and off. The light does not immediately come on after switching, but it takes a part of a second. Also when switching it off, it takes a small amount of time for the light to turn off.

It was already shown above that the magnetic flux is being build up due to a current in the conductor wire. In the story above, this all came together at the flux linkage (Equation 13 and 22):

$$\lambda = n \Phi \quad (25)$$

$$\lambda = L_m i \quad (26)$$

$$n \Phi = L_m i \quad (27)$$

When the current in the inductor is varied, the flux will thus change as well and this changes the electro-motive force e (EMF) (this will be shown in the next subsection of Faraday's law). The EMF is always resisting increasing current and resisting reducing current, like a mass resists acceleration. The work done (and thus the energy of the EMF) per time period (and thus actual power) against the back EMF is:

$$\frac{dW}{dt} = |e| I \quad (28)$$

where e is defined as:

$$e = - \frac{d}{dt} \lambda(t) = -n \frac{d}{dt} \Phi(t) \quad (29)$$

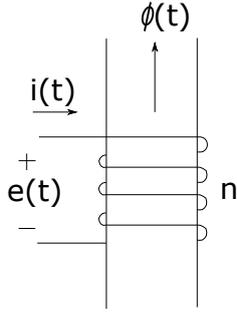


Figure 6: Voltage polarity and direction of flux and current in Faraday's law. Note the right hand rule between the change in flux due to the change in current.

assuming the number of turns n is time independent. To get to the energy W (in $[J]$):

$$W = \int_0^I dW \quad (30)$$

$$W = \int_0^I L I dI \quad (31)$$

$$W = L \int_0^I I dI \quad (32)$$

$$W = \frac{1}{2} L I^2 \quad (33)$$

For a structure without air gap, the inductance of Equation 24 is to be inserted in Equation 33 and the current from Ampere's Law (Equation 4):

$$W_m = \frac{1}{2} \frac{n^2}{\left(\frac{l_m}{\mu_m A_m}\right)} \left(\frac{l_m H_m}{n}\right)^2 \quad (34)$$

$$= \frac{1}{2} \frac{B_m^2}{\mu_m} A_m l_m \quad (35)$$

Where $A_m l_m$ is a volume and that in the linear region the flux density is described by $B_m = \mu_m H_m$.

The energy density in the core is then:

$$w_m = \frac{W_m}{A_m l_m} = \frac{1}{2} \frac{B_m^2}{\mu_m} \quad (36)$$

For structures with air gaps, the energy is primarily stored in the air gaps, as shown by the relative permeability $\mu_r = \mu_m/\mu_0$ and can be in the thousands. Therefore, it can be assumed that:

$$w = \frac{1}{2} \frac{B^2}{\mu} \approx \frac{1}{2} \frac{B_g^2}{\mu_0} \quad (37)$$

1.4 Faraday's law and Lenz's law

Faraday's law describes the induced voltage $e(t)$ in a coil due to the time-rate of change of the flux linkage λ , for a coil where the flux linkage makes an angle of 90° with all

windings. See also Figure 6³:

$$e(t) = -\frac{d}{dt}\lambda(t) = -n\frac{d}{dt}\Phi(t) \quad (38)$$

The polarity (minus sign) of the equation above are yet to be justified below. The original Faraday's law does not have the minus sign, but the minus sign is due to Lenz's law, explained below.

The relation of Equation 38 is valid, independent on the originator of the flux change. This means it is also possible when another coil on the same core influences the flux. This second coil can have:

- A time-variant current, or AC current;
- A fixed voltage $e(t)$ across the coil of Figure 6, causing a change in flux.

When integrating both sides of Faraday's law $e(t) = n\frac{d}{dt}\Phi(t)$ with respect to time, the following is found:

$$\Phi(t) = \Phi(0) + \frac{1}{n} \int_0^t e(\tau) d\tau \quad (39)$$

where $\Phi(0)$ is the initial flux at $t = 0$ and τ is an integration variable.

Above, at Figure 6, the right hand rule is shown between the change in flux and current direction. The thumb gives the direction of the current, and the fingers the direction of the flux change.

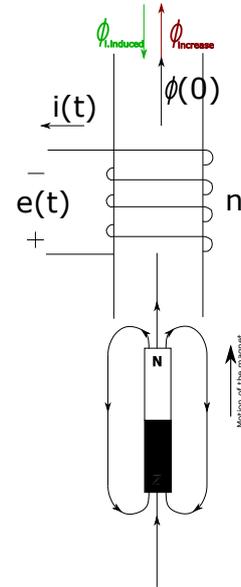


Figure 7: Example of a moving magnetic field (moving magnet) inducing a current in the coil. The direction is explained by the increase in flux, which needs to be countered by the flux due to the current. The flux due to the current and the direction of the windings determines the direction of the current. Note that once the magnet stops moving, the induced current also becomes zero.

³Assuming that the amount of windings is not time dependent, which is often the case.

However, Lenz's law, derived from the equation of energy, dictates that when a *flux increase introduces a current change*, the current *will flow the other direction*. If this was not the case, the flux increase would introduce a higher current, which introduces a higher flux change, and so on to infinity [1]. In short, this means that the response of current will always try to keep to flux equal. So if the flux increases, the direction of the induced current is such that its flux is in opposite direction of the flux increase. Note that a current increase will increase the flux as described in Equation 8.

This all sounds very theoretical, so let's take a look at an example as per Figure 7. There is a magnet, some core material and a wire wound around the core. The magnet is moving, and therefore the magnetic field is moving. This means that at the spot of the coil, the field is changing, and thus that the flux is increasing. As the magnet moves closer to the coil with the North pole, the flux density B increases, and thus the flux $\Phi(0) + \Phi_{increase}$ increases. As the law of Faraday dictates, this induces a current $i(t)$ in the wire, and as already known, the current will induce in turn a flux $\Phi_{i.induced}$ in the core.

To fulfill the law of conservation of energy, the flux increase $\Phi_{increase}$ and the induced flux $\Phi_{i.induced}$ must be opposite, and this is the result of Lenz's law. This means that the induced flux $\Phi_{i.induced}$ is opposite to an increasing flux, as shown in Figure 7. This means that the current will go opposite than as seen in Figure 6.

When the flux decreases, for instance when the magnet is moving away or when the magnet's South pole is facing the coil and the magnet is moving towards the coil, the $\Phi_{increase}$ is negative and will thus point in opposite direction, meaning that the $\Phi_{i.induced}$ is also negative and also points in the other direction.

1.4.1 Relating voltage, flux and current

When a current induces a flux in the core, the voltage $e(t)$ results in a flux $\Phi(t)$. As shown in Equation 27, it is not necessary to go back to Ampere's law to calculate the flux and flux linkage. The flux is directly related to the current:

$$\Phi(t) = \frac{L}{n} i(t) \quad (40)$$

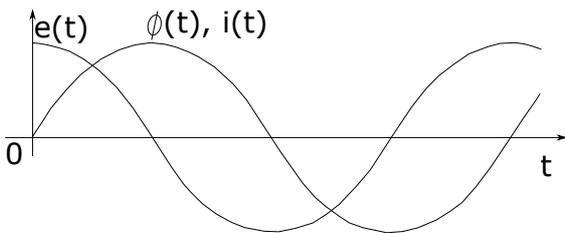


Figure 8: Relation between voltage e , flux Φ and current i for a sinusoidal voltage signal, inducing a current and thus flux in a core.

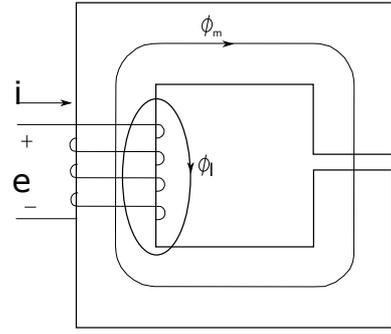


Figure 9: Representation of flux leakage.

Substituting this in Faraday's law, this results in:

$$e(t) = n \frac{d\Phi}{dt} = L \frac{di}{dt} \quad (41)$$

The relation results in a graph as shown in Figure 8 for a sinusoidal voltage input signal.

1.4.2 Leakage and magnetizing inductance

Similarly to conductors guiding currents in electric circuits, magnetic cores guide flux in magnetic circuits. Important difference is that the conductivity of copper in electric circuits is approximately 10^{20} times larger than air⁴, while the permeability of the magnetic core is in the order of 10^4 times as large as air. This makes flux leakages a bigger issue than current leakages in electric circuits. This is shown in Figure 9. The 'window' within the core is normally kept as small as possible, by designing a tight fit of the core in the coil.

The leakage is difficult to accurately estimate and often requires finite element analysis. However, it can be accounted for in the calculations and approximations can be made. The flux Φ is now divided into the magnetic flux (core flux) Φ_m , going through the coil (perpendicular to the coil) and through the core (and possible the air gap in the core), and the leakage flux Φ_l , going through the coil (perpendicular to the coil) and then partially through the air.

$$\Phi = \Phi_m + \Phi_l \quad (42)$$

As all flux goes *through the entire coil* (assumption), this can be rewritten to:

$$\lambda = n \Phi = n \Phi_m + n \Phi_l = \lambda_m + \lambda_l \quad (43)$$

$$\frac{\lambda}{i} = \frac{\lambda_m}{i} + \frac{\lambda_l}{i} \quad (44)$$

$$L_{self} = L_m + L_l \quad (45)$$

where L_{self} is the self-inductance, which is the total inductance.

With the equation $\lambda = (L_m + L_l) i$, Faraday's law can be rewritten to:

$$e(t) = L_l \frac{di}{dt} + L_m \frac{di}{dt} = L_l \frac{di}{dt} + e_m(t) \quad (46)$$

⁴This is why the leakage currents can be neglected at DC or low frequencies up to 60Hz for sufficient diameter cables.

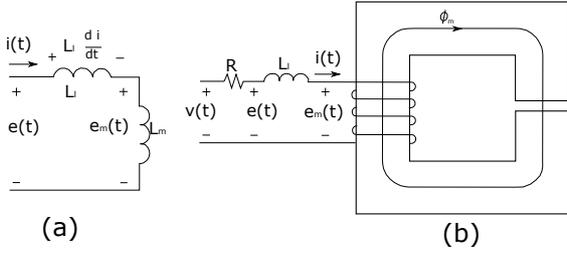


Figure 10: (a) Circuit representation of the flux leakage (b) Leakage inductance separated from the core.

The rewritten Faraday's law equation results in a circuit as shown in Figure 10, where the voltage drop due to the leakage inductance is shown separately so the voltage induced in the coil is solely due to magnetizing flux. The coil resistance R is then added to complete the coil representation.

Just for an approximation: Inductance leakages in transformers up to 2500kVA and windings reactance to windings resistance ratio between 3 and 6, the inductance leakage is between the 3% and 6%.

1.4.3 Mutual inductance

Electric machines often consist of several mutually coupled coils, where the flux of one coil influences the other. When the EMF is induced in the same circuit as where the current is changing, this effect is called self-inductance, as shown above.

When the EMF is induced by an adjacent coil within the same magnetic field, it is called mutual inductance M (in H or Henry).

The mutual coupling can be a desired effect, for instance for transformers, motors or generators. However, it can also have a negative influence, leading to leakage inductance that can interfere with the other component, which would need protection against the interference by electrical screening or ground potential.

The amount of mutual inductance M depends on:

- The distance between the coils l ($[m]$), as this changes the flux Φ through the secondary spool: The smaller the distance, the more interference;
- Medium for the flux μ_r or μ_0 ($[H/m]$), as this influences the flux Φ through the secondary spool: Two coils wrapped around the same soft iron core will have more mutual inductance than in air;
- Cross section of the core A ($[m^2]$): The larger the cross section of the core, the larger the mutual inductance;
- The amount of turns n ($[-]$), as the amount of turns of the primary spool influences the flux Φ and the turns of the secondary spool influences the induced current in the second spool: The more turns, the greater the mutual inductance.

The Electro-Magnetic Force (EMF, e) is already shown in Equation 29. Now let coil 1 have a changing current i_1 , resulting in a changing flux Φ_{21} in coil 2. This then leads to:

$$e_{21} = -n_2 \frac{d\Phi_{21}}{dt} = -\frac{d}{dt} \int \int_{coil2} B_1 dA_2 \quad (47)$$

Where e_{21} (in $[V]$) is the EMF, n_2 (in $[-]$) is the amount of windings of the secondary coil⁵, Φ_{21} (in $[Wb]$) is the changing flux in the secondary coil due to changing current in the primary coil, B_1 (in $[T]$ or $[Wb/m^2]$) is the magnetic field due to coil 1 and A_2 (in $[m^2]$) is the coil area.

With the EMF defined as above, it is possible to define the mutual inductance⁶:

$$n_2 \frac{d\Phi_2}{dt} = M \frac{di_1}{dt} \quad (48)$$

Note that the mutual inductance M is proportional to the flux and current:

$$M = \frac{n_2 \Phi_2}{i_1} \quad (49)$$

With the general Equation 47 the influence of two coils / wires on each other can be estimated. There are two examples shown in Figure 11. The first is the influence of two concentric wires on each other through air, the other two coils on the same core.

Example 1: Concentric wires in air: For the first example (Figure 11 (a)), the flux density of wire 1 B_1 ($[T]$) is dependent on (note that $n_1 = 1$):

$$B_1 = \frac{\mu_0 i_1}{l_m} = \frac{\mu_0 i_1}{2 R_1} \quad (50)$$

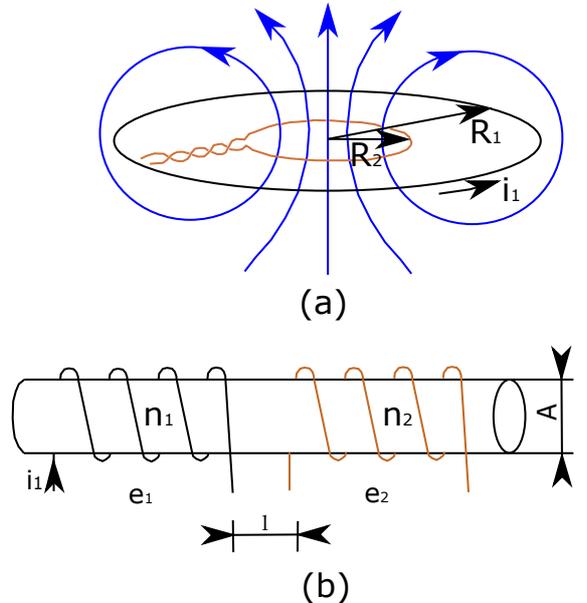


Figure 11: Examples of mutual conductance on a single core

⁵Secondary means it has current changes due to changes in the flux, while the primary coil has a changing current resulting in changing flux

⁶Here the definition of $M_{12} = M_{21} \equiv M$ is already applied

Since $R_1 \gg R_2$, so the distance between the wires is not close to zero, the approximation is made that the field density in the core is homogenous. Therefore:

$$\Phi_{21} = B_1 A_2 = \left(\frac{\mu_0 i_1}{2 R_1} \right) \pi R_2^2 = \frac{\mu_0 \pi i_1 R_2^2}{2 R_1} \quad (51)$$

Now using the definition of the mutual inductance:

$$M = \frac{\Phi_{21}}{i_1} = \frac{\mu_0 \pi R_2^2}{2 R_1} \quad (52)$$

Example 2: Two coils on a single core: Now for the second example, Figure 11 (b), where two coils wrapped around a single core. The flux density, flux and mutual inductance become:

$$B = \mu_m \frac{n_1 i_1}{l_1} = \mu_r \mu_0 \frac{n_1 i_1}{l_1} \quad (53)$$

$$\Phi = B A = \frac{\mu_r \mu_0 n_1 i_1}{l_1} A \quad (54)$$

$$M = \frac{\mu_r \mu_0 n_1 n_2 A}{l_1} \quad (55)$$

Where M (in $[H]$) is the mutual inductance, μ_r ($[-]$) is the relative permeability, μ_0 ($4\pi \cdot 10^{-7} [H/m]$) is the permeability of air, n_1 and n_2 ($[-]$) are the number of windings of the coils, A (in $[m^2]$) is the cross-section of the coil influenced by the EMF and l_1 (in $[m]$) is the length of coil 1.

By using the inductance of electromagnets, the mutual inductance can be coupled to the inductance of electromagnets and a coupling factor. First to define the self inductance of electromagnets:

$$L_1 = \frac{\mu_0 \mu_0 n_1^2 A}{l} \quad (56)$$

$$L_2 = \frac{\mu_0 \mu_0 n_2^2 A}{l} \quad (57)$$

Now the mutual inductance is:

$$M^2 = L_1 L_2 \quad (58)$$

$$M = \sqrt{L_1 L_2} \quad (59)$$

The equation above is only valid if all the flux going through the secondary coil. If this is not the case, the coupling factor k is defined as:

$$k = \frac{M}{\sqrt{L_1 L_2}} \quad (60)$$

$$M = k \sqrt{L_1 L_2} \quad (61)$$

1.5 Practice: Induction in sensor cables

The previous section shows that a current in a wire does not only create a magnetic field, but the changing magnetic field can cause a current in a cable. This can distort the signal of the sensor to the input of the PLC, leading to measurement errors. The most common ways to prevent this, are:

1. Keep sensor cables far away (200mm) from power cables on the cable trays;
2. Use earthed cabled (outside of the cable connected to earth, minimizing the effect on the actual sensor cable, similar to the shield of the cable between PPU and motor, as shown in [3]);
3. Use twisted pair sensor cables;
4. Use 4-20mA signal instead of +/-10V signal, as the sensor then compensates the extra current.

See also Section 4 for another example to filter high frequency noise from signal cables. Another way is using filters as shown in [3].

1.6 Transformers

Mutually coupled coils are widely used in electric machines. A transformer is an example and exists of two or more tightly coupled windings where (almost) all flux produced by one winding links with the other winding(s).

1.6.1 Ideal transformer

To start simple, lets assume that coil 1 (primary coil) has no resistance and the leakage inductance is zero (meaning that all created flux in coil 1 will go through the core and through the other windings), as shown in Figure 12 (a) and (b). Faraday's law then leads to:

$$e_1(t) = -n_1 \frac{d\Phi_m}{dt} \quad (62)$$

or

$$\Phi_m(t) = -\frac{1}{n_1} \int_0^t e_1(\tau) d\tau \quad (63)$$

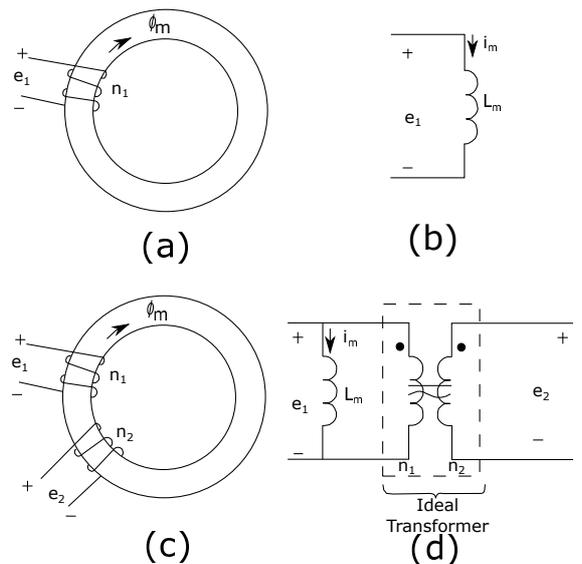


Figure 12: Inductance of one and two coils with the equivalent circuit diagrams

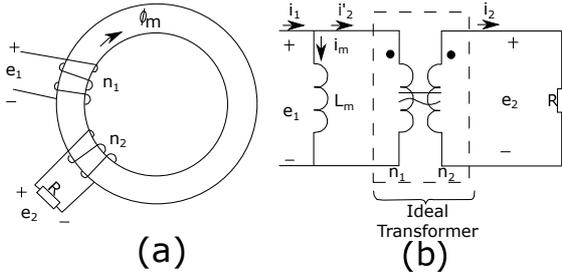


Figure 13: The secondary circuit is closed using a resistor

The current $i_1(t)$ drawn to create the flux Φ_m depends on the magnetizing inductance L_m , as shown for a coil (Figure 12 (b)).

Now the core has a second coil, as shown in Figure 12 (c) and (d). The second coil has n_2 windings and has an induced voltage due to the changing flux:

$$e_2(t) = -n_2 \frac{d\Phi_m}{dt} \quad (64)$$

When comparing Equation 62 and Equation 64, it is shown that the Volts per turn are the same due to the same $d\Phi_m/dt$, and thus:

$$\frac{e_1(t)}{n_1} = \frac{e_2(t)}{n_2} \quad (65)$$

$$\frac{e_1(t)}{e_2(t)} = \frac{n_1}{n_2} \quad (66)$$

The above equations are known as the **ideal transformer**. In Figure 12 two dots are drawn at the positive sides of the coils. This means that if flux Φ_m increases, the voltages at these two terminals will be positive with respect to the terminals without dots. This has the advantage that the orientation of the coils does not need to be plotted.

Now a resistor R is added to close the secondary circuit, as shown in Figure 13, meaning that there is now a current i_2 in the secondary circuit. Due to the Ampere-turns $n_2 i_2$, the current i_2 will tend to influence the the flux Φ_m , but physically the flux Φ_m is only dictated by the voltage e_1 , as shown in Equation 62. This means that the current through coil 1 is compensated for the effect of the current in coil 2, as shown in Figure 13:

$$i_1(t) = i_m(t) + i_2'(t) \quad (67)$$

$$n_1 i_2' = n_2 i_2 \quad (68)$$

$$\frac{i_2'}{i_2} = \frac{n_2}{n_1} \quad (69)$$

This is the second property of an ideal transformer: The total current drawn from the terminals of coil 1 is equal to the current to create the flux plus the current required to compensate for the flux of the second coil.

1.6.2 Core and electrical losses

The first difference between the real world and an ideal transformer, is the resistance and leakage inductance associated with coil 2. As shown in Figure 14, these can be

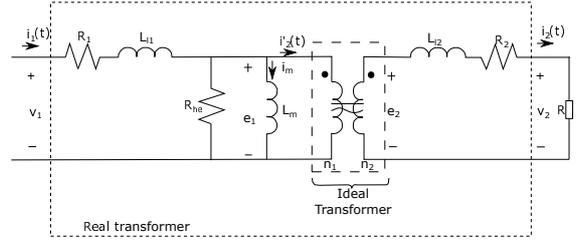


Figure 14: The model of a real transformer

modeled in series with the R-L load. This means that the induced voltage e_2 is different from the voltage at the terminals v_2 . Similarly, at coil 1 the EMF voltage e_1 is different from applied voltage v_1 .

The core losses and the B-H hysteresis are modeled by connecting a resistance $R_{h,e}$ parallel to L_m .

Another source of core losses are eddy currents, also called circulating currents in the magnetic core. All magnetic materials have a finite electrical resistivity, while ideally it would be infinite. At Faraday's law it was already seen that flux then thus results in a potential (so voltage) difference within the core material. This is also shown in Figure 15. Eddy currents can be minimised by making the core laminated, and each layer is electrically isolated from the other. The laminations are perpendicular to the shaft axis. This way the area for the Eddy currents to run over are smaller, making the losses smaller, as this reduces the loops in which the current can flow. The lamination does however reduce the flux as well, although the losses are more effected than the flux (thus resulting in nett lower losses). For 50Hz - 60Hz lamination thicknesses are typically 0.2 to 1mm.

1.6.3 Determine real transformer parameters

To determine the model parameters as shown in Figure 14, there are two types of tests to do: The open-circuit test and the short-circuit test.

Open-circuit test The open-circuit test leaves one of the coils, for instance coil 2, open. This means that the circuit is not closed. At coil 1 the voltage is applied as done normally. To calculate the impedance of the spool, the RMS values for the input voltage V_{oc} , current I_{oc} and the average (real) power S_{oc} are measured. The subscript oc here stands for 'open circuit'.

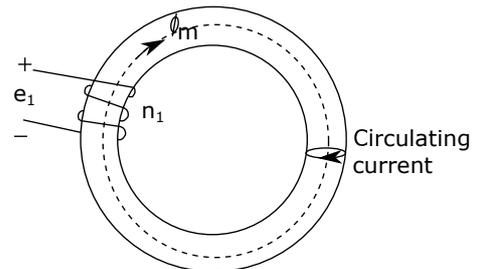


Figure 15: Eddy current induced in the magnetic core

In the open-circuit condition, the secondary coil will not drain much energy, meaning that i_2 is (almost) zero, meaning that (almost) all current will go through the spool L_m and the resistor R_{he} . This also simplifies the model, as shown in Figure 16 (a).

When calculating the impedance of the resistor R_{he} , the formula of [5], paragraph 2.4.2, Equation 72 can be used:

$$R_{he} = \frac{V_{oc}^2}{S_{oc}} \quad (70)$$

The magnitude of the open loop impedance is (as shown in [5] paragraph 2.2):

$$|Z_{oc}| = \frac{X_m R_{he}}{\sqrt{R_{he}^2 + X_m^2}} = \frac{V_{oc}}{I_{oc}} \quad (71)$$

In the equation above, only the X_m is unknown and therefore it can be solved.

Short-circuit test In the short-circuit test, one circuit, in this case the primary circuit, is short-circuited. This is shown in Figure 16 (b). Due to the short-circuit, the inductance L_m and resistance R_{he} can be neglected, as the current will flow much easier through the short-circuit.

Just as for the open-circuit test, the RMS voltage V_{sc} , RMS current I_{sc} and (real) power S_{sc} are measured. From Equations 66 and 69, the following can be concluded:

$$\frac{e_2}{e_1} = \frac{n_2}{n_1} \quad (72)$$

$$\frac{\bar{I}_{sc}}{\bar{I}_1} = \frac{n_1}{n_2} \quad (73)$$

$$\frac{e_2}{\bar{I}_{sc}} = \left(\frac{n_2}{n_1}\right)^2 \frac{e_1}{\bar{I}_1} \quad (74)$$

Looking at Figure 16 (b), notice that:

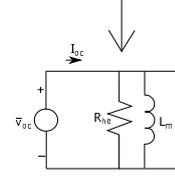
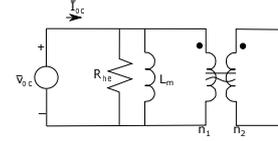
$$\frac{e_1}{\bar{I}_1} = R_1 + L_{l1} = R_1 + j X_{l1} \quad (75)$$

$$\frac{e_2}{\bar{I}_{sc}} = \left(\frac{n_2}{n_1}\right)^2 (R_1 + j X_{l1}) \quad (76)$$

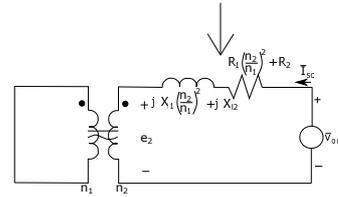
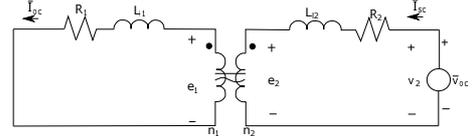
This means that the resistor and spool of the primary circuit can be modeled as part of the secondary circuit, simplifying the model as shown in Figure 16 (b). This reduces the measured quantities to:

$$R_2 + \left(\frac{n_2}{n_1}\right)^2 R_1 = \frac{S_{sc}}{I_{sc}^2} \quad (77)$$

The windings of the two coils in a transformer are chosen such that the $I^2 R$ losses in the coils are approximately equal. These losses are due to the losses in the copper. This means that the resistance of each coil is proportional to the square root of the rated current. Looking at Equation 69, where



(a)



(b)

Figure 16: (a) The open-circuit test. (b) The short-circuit test

$i_m = 0$ and thus $i_2 = i_1$, the following is obtained:

$$\frac{i_1}{i_2} = \frac{n_2}{n_1} \quad (78)$$

$$\frac{R_1}{R_2} = \left(\frac{i_2}{i_1}\right)^2 = \left(\frac{n_1}{n_2}\right)^2 \quad (79)$$

$$R_2 = R_1 \left(\frac{n_2}{n_1}\right)^2 \quad (80)$$

When substituting the result above in Equation 77, R_2 can be calculated, which means that R_1 is calculated as well:

$$R_2 + R_2 = 2 R_2 = \frac{S_{sc}}{I_{sc}^2} \quad (81)$$

$$R_2 = \frac{1}{2} \frac{S_{sc}}{I_{sc}^2} \quad (82)$$

This leaves the spool, which is now the spool of the primary circuit in series with the spool of the secondary circuit. The two spools are now in series, meaning (see [5] Paragraph 2.2 and Equation 76 above):

$$Z_{Leq} = j X_{l2} + j X_{l1} \left(\frac{n_2}{n_1}\right)^2 \quad (83)$$

The leakage reluctance in a winding is approximately pro-

portional to the square of its turns:

$$\frac{X_{l1}}{X_{l2}} = \left(\frac{n_1}{n_2}\right)^2 \quad (84)$$

$$X_{l2} = X_{l1} \left(\frac{n_2}{n_1}\right)^2 \quad (85)$$

$$Z_{Leq} = jX_{l2} + jX_{l2} = 2jX_{l2} \quad (86)$$

For the entire loop as shown in the lowest (simplified) circuit diagram of Figure 16, the impedance becomes:

$$Z_{sc} = 2R_2 + 2jX_{l2} \quad (87)$$

$$|Z_{sc}| = \sqrt{(2R_2)^2 + (2X_{l2})^2} = \frac{V_{sc}}{I_{sc}} \quad (88)$$

With the impedance above, combined with the value R_2 already known and the measured quantities V_{sc} and I_{sc} , the spool is calculated as well, meaning that all values can be calculated.

1.7 Permanent magnets

Up till now all flux was created by coils, meaning that each coil also has $i^2 R$ losses. Especially higher power ratings or higher power density applications will start to use permanent magnets in the magnetic circuits, to boost the total flux in the magnetic circuit.

As shown in Figure 17, especially $Nd - Fe - B$ magnets offer a high flux operation. This material will, due to innovations in manufacturing and operation in high temperatures, become more and more present in circuits. In the following section the permanent magnets will be used in magnetic circuits.

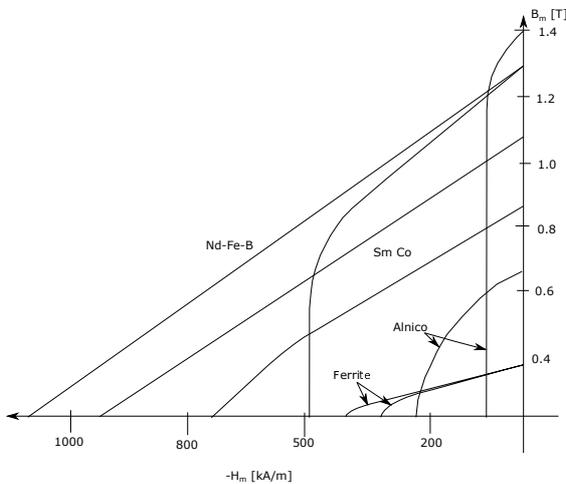


Figure 17: Different materials for permanent magnets

2 Electro-Mechanical energy conversion

Electric motors⁷ convert electrical power to mechanical power, but may also work the other way around. This is not only true for generators, but is true for electric motors that are slowing down (regenerative braking) as well.

Electric motors have a stator (static outer part), an air gap and a rotor. To reduce the amount of Ampere-turns to create the flux, both the stator and rotor are made of high permeability ferromagnetic materials and the air gap is kept small. Typically, (for motors under 10kW) the air gap will be approx 1mm. As this air gap is so small, for an approximation the radius at the middle of the air gap can be used in calculations.

The motor can have 2 or more (but even amount of) poles. The amount of poles are equal to the amount of North and South poles the motor has. There are motors commercially available of more than 12poles, although the most commonly used are the 2, 4 and 6 pole motors⁸. The more poles a motor has, the slower the motor will rotate, as the angle it needs to rotate for one period becomes smaller. Due to the symmetry of the multiple-pole motors, it is often only required to consider one pole, consisting of one North pole and the adjacent South pole.

When the stator and rotor are perfectly round, as assumed above, the motor is called a non-salient motor. This means that the air gap is uniform and thus the magnetic fields are the same for each pole. There are machines or motors where the magnetic fields are not equal by design, which are called salient drives, and is shown in the bottom of Figure 18. Salient motors will not be explained in depth here, just a few characteristics:

- Salient drives are normally allowed at lower speeds (100rpm - 1500rpm);
- Salient drives often require damper windings to avoid oscillations;
- Hydropower plants is one example where salient drives are often found;
- The diameter of salient drives are often larger, while the length is shorter.

2.1 Production of magnetic field

Now to apply the theory of the previous section to electric motors. In Figure 19 (a) the cross-section of a 2-pole motor is shown, where all electric circuit of the rotor is not shown and the circuit of the stator is simplified. The magnetic flux lines are shown. For this first approach, the leakage flux is neglected.

The flux (Φ) lines in the air gap are radial, meaning that the field intensity in the air gap is also radial (shortest length

⁷Here only motors are shown, linear drives have similar principles

⁸Based on ABB motor catalogue.

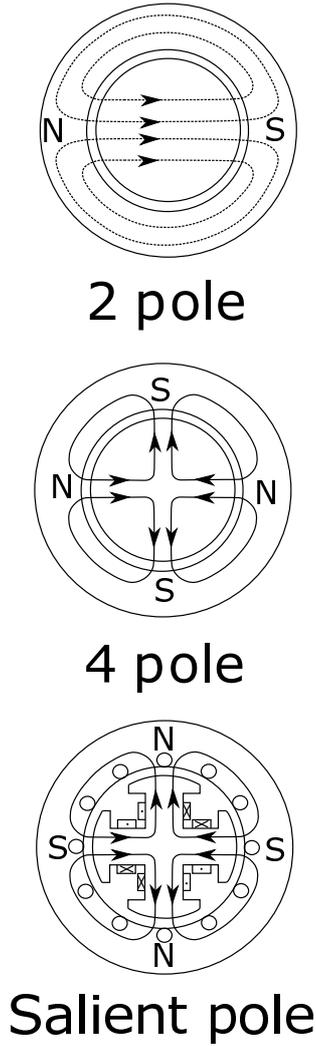


Figure 18: The structure of the 2, 4 and salient pole motor

possible to cross the air gap). The field intensity is considered positive ($+H_s$) if it is away from the center and negative ($-H_s$) towards the center. The subscript 's' here means 'stator'.

As a simplification, the permeability of iron will now be assumed to be infinite, meaning that the reluctance of the iron is negligible and the reluctance of the air gap is dominating. Now looking at the field lines and using Ampere's law (note that the field is positive one way and negative the other):

$$H_s l_g - (-H_s) l_g = N_s i_s \quad (89)$$

$$H_s = \frac{N_s i_s}{2 l_g} \quad (90)$$

As the permeability of stator and rotor is assumed infinite, there is no 'resistance', meaning that half the Ampere-turns $N_s i_s/2$ is used to make the flux lines across the air gaps, while the other half is used to create the Magneto-motive

force (MMF) F_s :

$$F_s = \frac{N_s i_s}{2} \quad (91)$$

$$F_s = H_s l_g \quad (92)$$

$$B_s = \mu_0 H_s = \mu_0 \frac{N_s i_s}{2 l_g} \quad (93)$$

where all quantities are assumed positive when directed away from the center.

Figure 19 b) shows the radial view of the motor, as if the circle was cut and made into a straight line. The flux density B is just as the field H_s and MMF F_s dependent on the instantaneous current $i_s(t)$, meaning that the Figure 19 b) only shows these values at one point in time. This means that the square B_s in the Figure also represents H_s and F_s .

2.2 Basic principles of operation

There are two basic principles that govern AC electric machines to convert electrical energy to mechanical work:

1. A force is produced on a current-carrying conductor, while it is subjected to an external electric field;
2. An EMF is induced in a conductor moving in a magnetic field.

Both these principles will be further explained in the subsections below.

2.2.1 Electromagnetic force / synchronous motor

Consider a conductor of length l as shown in Figure 20, which is carrying a current i and is subject to an external established magnetic field of a uniform density B perpendicular to the conductor length. A force of f_{em} is exerted on

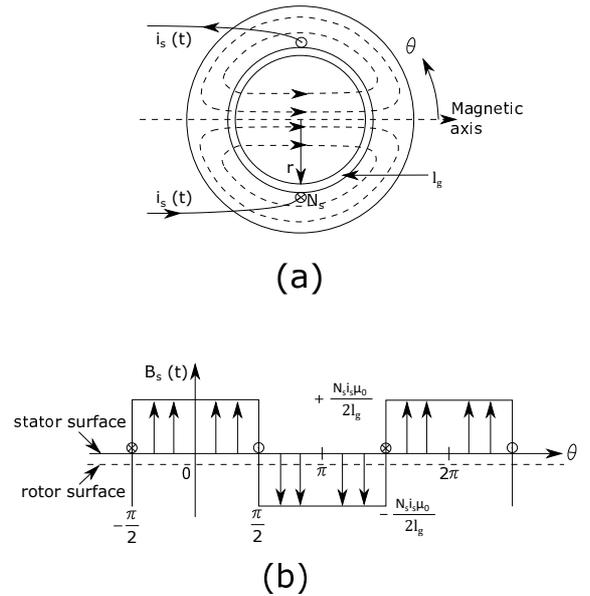


Figure 19: The production of a magnetic field in a 2 pole (induction) motor

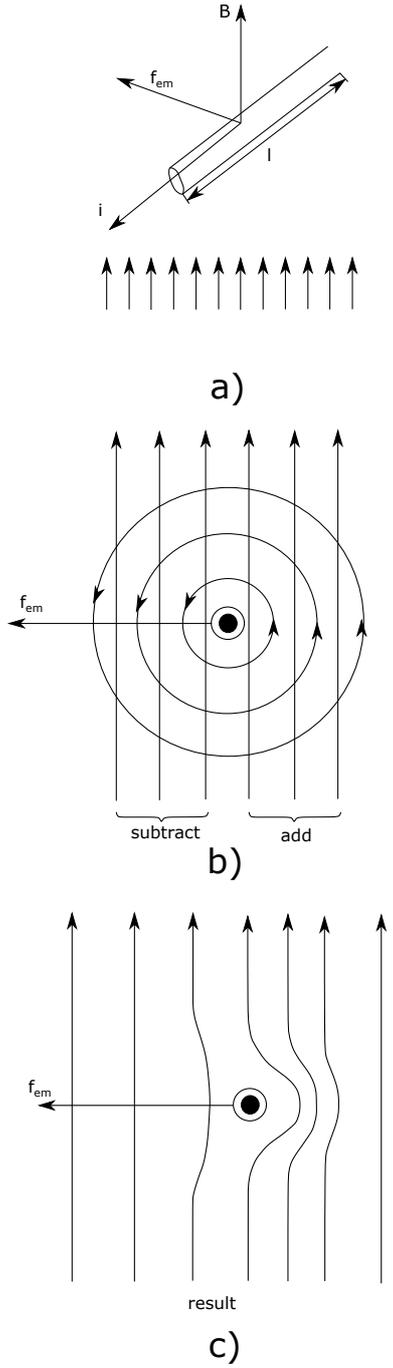


Figure 20: Electric force on a current carrying conductor in a magnetic field

the conductor due to electromagnetic interaction between the magnetic field and the conductor current:

$$f_{em} = B n i l \quad (94)$$

Note here that the force f_{em} is in $[N]$, as Tesla $[T] = [\frac{Wb}{m^2}] = [\frac{V \cdot s}{m^2}] = [\frac{kg}{s^2 A}] = [\frac{N}{A \cdot m}]$ and as voltage $[V] = [\frac{kg \cdot m^2}{s^3 A}]$.

The force f_{em} is perpendicular to the current i as well as the magnetic field density B . In order to obtain the direction of the force, the external magnetic field and the produced magnetic field due to the current through the conductor

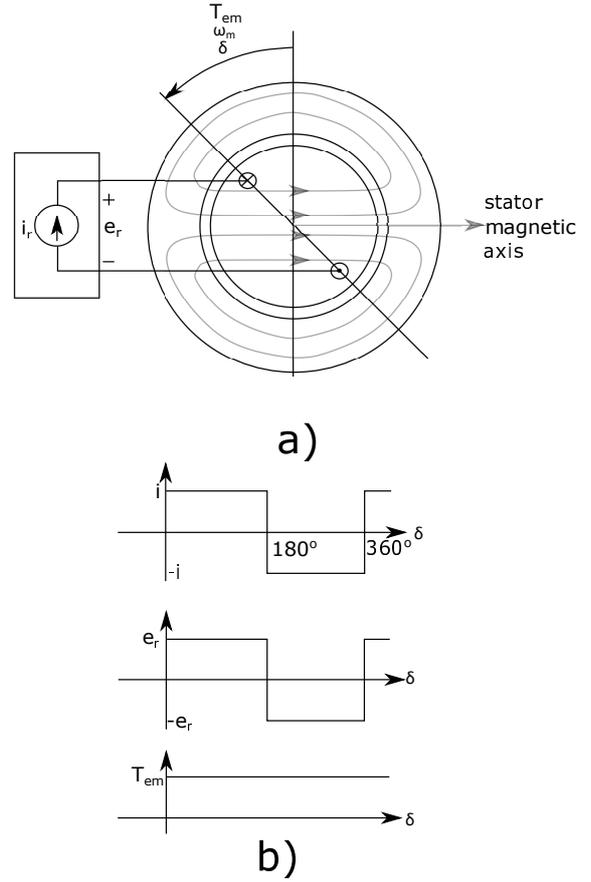


Figure 21: Example of a synchronous motor

need to be superimposed, as shown in Figure 20 b). The superposition leads to a higher density at one side, and a lower density at the other side, as shown in Figure 20 c). The force f_{em} will then point from the high density to the low density.

This force is then the same on the other side of the motor, although the current flows the other way. This means that the f_{em} will also point the other way, resulting in a torque on the rotor. This torque is then driving the motor en converting electrical energy (current in the wire) to mechanical work. It does however mean that once the wires are at each side of the motor, the torque decreases as the arm of the forces becomes small. Therefore, multiple wires can be used to make the motor rotate more stable.

In Figure 21 an example of the synchronous motor is shown. Assume that the motor has n windings. From above it can thus be concluded that:

$$f_{em} = B n i l \quad (95)$$

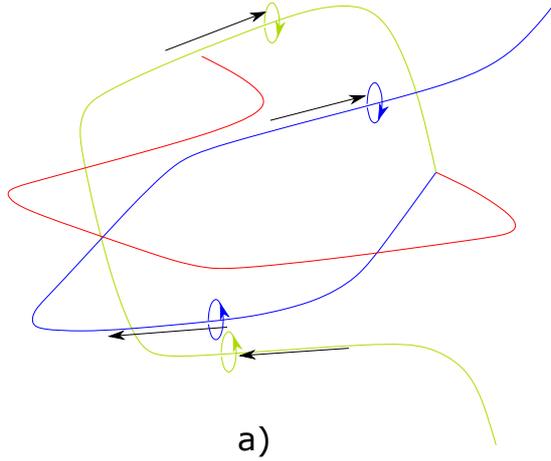
The torque is then:

$$T_{em} = 2 f_{em} r \quad (96)$$

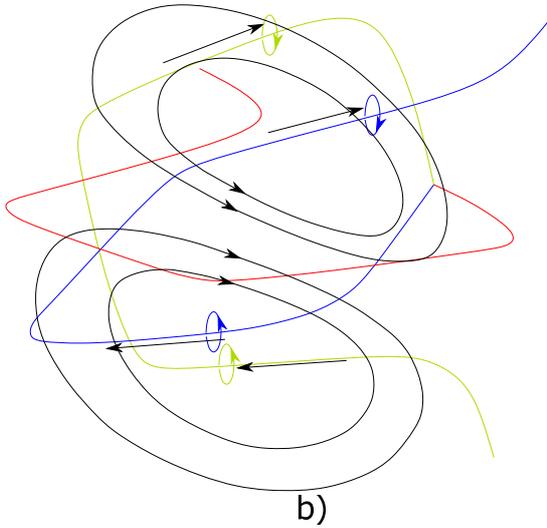
$$T_{em} = 2 B n i l r \quad (97)$$

where r (in $[m]$) is the radius of the windings.

As the motor turns, the direction of the current in the wires also needs to turn, to keep the torque in the correct direction. That the torque remains constant here is



a)



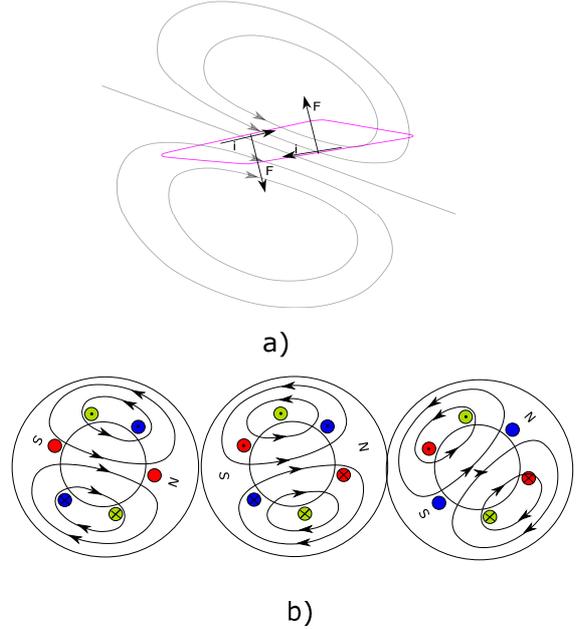
b)

Figure 22: The stator of the 3-phase induced motor, producing a rotating magnetic field

an approximation. Note that an *equal amount of torque is supplied to the stator*, meaning that the stator needs to be properly fixed.

2.2.2 Induced EMF / induction or asynchronous motor

To understand the induction motor, first the stator only must be examined, as the stator creates the rotating magnetic field. The stator is shown in Figure 22 a), where the three phases (green, blue and red) are shown. As all three phases are rotated 120° from each other, their current through the wires has the same phase shift. Mechanically the phases are also shifted 120° . Assume that in one particular moment in time the green line has no current. The current in the blue and red phases create a field around the wires, and by routing the wires these all create a magnetic field that is perpendicular to the wires. As the AC current will change in all wires, the magnetic field will rotate. The speed of the rotating magnetic field is called synchronous speed N_s .



a)

b)

Figure 23: a) The rotor, represented by a wire, in the rotating magnetic field, creating a current and thus Lorentz force that rotates the rotor. b) The rotating magnetic field and thus the induced current in the rotor.

The rotor has either a cage or windings, but no power source. In Figure 23 a) the magnetic field is rotating and will thus induce an EMF in the wire according Faraday's law (Equation 38), but only when the magnetic field and rotor have different speeds. This is because the voltage is only induced if there is a change in flux ($-n \frac{d}{dt} \Phi(t)$). The wire will then have a current due to the induced voltage. That current in a magnetic field will create Lorentz forces, as also shown in Figure 23 a). Note that the Lorentz forces are only generated when the speed of the rotor and rotating magnetic field are not equal. This means that the speed of the rotor is always smaller than the speed of the rotating magnetic field ($N_s > N_{rotor}$). This is why the induction motor is also called an **asynchronous motor**.

The difference between rotor speed and synchronous speed is called slip S :

$$S = \frac{N_s - N_{rotor}}{N_s} \quad (98)$$

As the magnetic field can be created by the stator and starts to rotate and produce a torque, an induction motor is self-starting. See also [2] for a video on the working of the induction motor.

For the calculation of the motor, see Figure 24 a), where a conductor with length l is moving with speed u through a flux density B (pointing into the paper). Now assume that the conductor is moving to the right (which can also mean that the field is moving to the left). Faraday's law

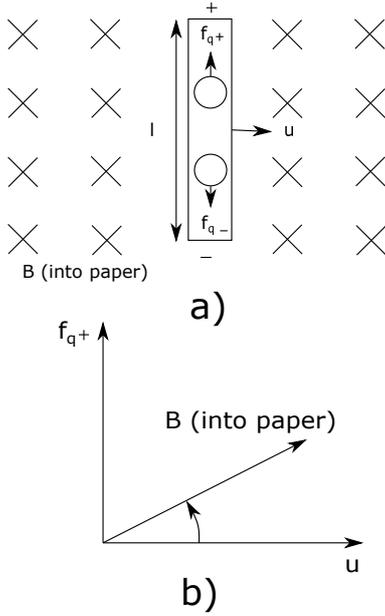


Figure 24: Conductor moving through a magnetic field

(Equation 38) then dictates (for an uniform B):

$$e(t) = -n \frac{d}{dt} \Phi(t) \quad (99)$$

$$e(t) = -n \frac{d}{dt} \int \int B dA \quad (100)$$

$$e(t) = -n B \frac{d}{dt} \int \int l dA \quad (101)$$

$$e(t) = -n B \frac{d}{dt} l x(t) \quad (102)$$

$$e(t) = -n B l u(t) \quad (103)$$

$$e(t) = -n B l r \omega_m \quad (104)$$

where r (in [m]) is the radius of the windings and ω_m (in [rad]) is the relative movement of the rotor and the magnetic field. And as $n = 1$, the induced voltage is (note that the minus sign only shows direction):

$$e(t) = -B l u(t) \quad (105)$$

$$(106)$$

Now the Lorentz force can be calculated. The electric force on the particles will be neglected ($F_e = qE = 0[N]$, where q is the electric charge of the particle in Coulomb and E is the electric field in [V/m]). The magnetic force is the governing Lorentz force:

$$\vec{F}_b = q (\vec{u} \times \vec{B}) \quad (107)$$

where q is again the charge of the particle (positive, or negative in the case of an electron) in Coulomb, \vec{u} is the vector of the speed of the particle, \vec{B} is the vector of the flux density in [T] and the Lorentz force \vec{F}_b (in [N]) is determined by the cross product of the speed and flux density vectors. This cross product means that the Lorentz force is perpendicular to the speed and flux density vectors. This is also shown in Figure 24 b).

The above is a calculated induced EMF without a current in the conductor. When there is a current in the conductor (as is the case for motors, to develop the Lorentz force), the voltage will be the induced EMF as shown above, minus the voltage drop across the conductor resistance and inductance.

The torque of an n -windings rotor will then be:

$$e(t) = -n B l r \omega_m \quad (108)$$

$$F_b = B I l \quad (109)$$

$$T_{em} = 2 n F_b r \quad (110)$$

where F_b is the force on one wire.

2.3 Energy conversion

In an electric motor, electric power is used in combination with magnetism to create mechanical power, or it is the other way around for regeneration. When excluding all losses, the following is valid:

$$P_{mech} = P_{el} \quad (111)$$

$$P_{mech} = T_{em} \omega_m \quad (112)$$

$$P_{el} = e_r i = 2 B n l r i \omega_m \quad (113)$$

The losses will be included in paragraph 2.3.2.

2.3.1 Regenerative braking

When a motor is rotating at a speed of ω_m , the rotor inertia and the inertia of the load (divided by gearbox ratio if applicable) has stored mechanical energy. This means that for stopping, the motor should have a torque with a direction opposite of the rotation direction. This can be done by regenerative braking, meaning that the current will flow in the opposite direction, as shown in Figure 25 b). Note that the voltage is still the same, so the electric power supply is not swapped.

Drawback of regenerative braking is the harmonic distortion on the power grid, also called the cosine ϕ shift, which is the shift between the voltage and current phases, as shown in [5] Section 2.3 Power. The phase shift is mainly due to the current used to magnetize the motor, where the motor can be seen as an inductor. A measure for the harmonic distortion is based on the higher n -th harmonics current I_n present in the current phase, compared to the first (fundamental) current I_1 :

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1} \quad (114)$$

Depending on the equipment in the power circuit, more or less distortion can be allowed. For example critical medical equipment require clean electrical energy with less than 5% harmonic distortion.

To start the mechanical and electrical power is examined:

$$P_{mech} = T \omega = T \frac{n}{60} 2 \pi \quad (115)$$

$$P_{elec} = U_{DC} I_{DC} = \sqrt{3} U_{AC} I_{AC} \cos(\phi) \quad (116)$$

In older systems the braking was either limited on the motor, so other brakes are installed to brake, or the power from the motor was fed to brake resistors (also called brake choppers), transforming the electrical power to heat and cooling the brake resistor with cooling water. An advantage is that the brake resistor works even if AC supply is lost. In newer systems the power can be fed back to the electrical grid. There are a couple of systems available in the industry, as will be shown below.

Thyristor bridge The thyristor bridge is made up of two sets of diodes, one set for driving the motor (forward in Figure 26 a)), the other for braking. Only one bridge operates at any point in time, the other is blocked. The switching is based on the DC voltage. It is a widely used and cost effective way to use regenerative braking, possible of continuous braking. One large drawback is that the DC voltage is lower than AC voltage, in order to keep some margins for control, which often then leads to the installation of a step-up transformer. Furthermore, the phase difference ϕ is dependent on the load, meaning this solution can cause harmonic distortion in the AC grid. Also, braking is not available during a power loss.

Regenerative braking unit Another way to feed the energy back to the system, is by using a regenerative braking

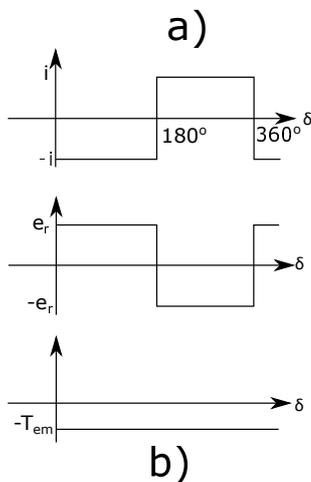
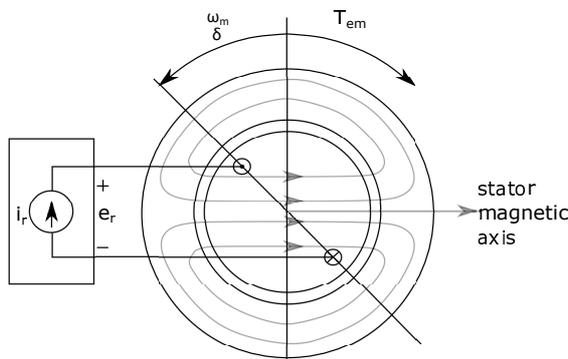


Figure 25: Regenerative braking

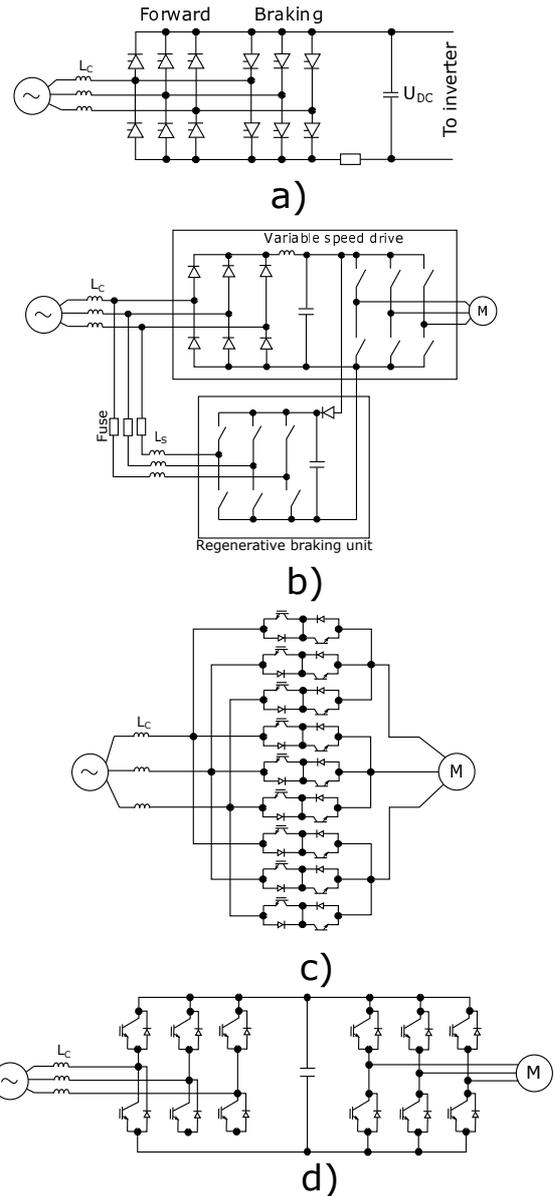


Figure 26: a) Thyristor bridge. b) Matrix for regeneration. c) Regenerative braking unit. d) IGBT bridge.

unit, as shown in Figure 26 b). It uses thyristors or IGBTs to connect to the correct power line when the DC capacitor is charged due to braking. Advantage is that this unit can be optimised for the braking requirements, minimizing distortions, and has lower harmonic distortions than the thyristor bridge. It does however often require a filter to limit the angle ϕ between voltage and current and additional fuses to secure the AC supply.

Matrix configuration In Figure 26 c) the matrix technology is shown. This uses nine b-directional IGBTs to be able to connect each line of the motor to each AC power line. This means there is no DC bus capacitor. By connecting each line of the motor to each power line, the voltage to the motor can be controlled at the proper frequency. By controlling the voltage, braking is also possible.

The advantage is that the amount of harmonic disturbances are lower than the previous two solutions and due to the single component it takes less volume than the brake resistor. It is however more expensive than the previous versions and has high demands on the supplied voltage, as it otherwise can become unstable. The maximum output voltage is also approximately 87% of the AC voltage to keep the harmonic distortions low, otherwise the harmonic disturbances increase significantly. The switching frequency requires some filtering to suppress the distortion. This braking method cannot be used during a power loss.

IGBT bridge In Figure 26 d) the IGBT bridge is shown. It uses twelve IGBT, six to control the AC to DC and six to control the DC to motor power. The control of the IGBT needs to keep three general targets: Keep DC voltage stable, minimise supply current (and thus $\cos(\phi) \approx 1$) and minimise harmonic distortion. It is possible to brake continuous, this solution has the lowest harmonic disturbance shown in this article and the DC voltage does not need to be lower than the AC voltage (it can even be boosted). By reducing ϕ the power factor is influenced, as shown in [5]. It is however more expensive than all other solutions and it needs filtering for the switching frequencies and of the neutral phase. This braking method can also not be used during a power loss.

Common DC Another option is to connect multiple motors to the same DC bus, and if one motor is braking, the other consumes the extra energy. It often, but not always, needs a brake resistor for security. This is however quite dependent on the mechanical system whether this is possible. The required motor power should at all times be equal or higher than the braking power.

2.3.2 Power losses and efficiency

For a frequency drive, there are three main losses that need to be taken into account:

- Motor losses;
- Power Processing Unit (PPU) losses;
- Mechanical losses.

The mechanical losses are neglected here, as these are dependent on the mechanical design.

The relationship of the nett delivered electrical power P_0 (so without mechanical losses) and the delivered electrical power $P_{inmotor}$ are:

$$P_{inmotor} = P_0 + P_{losses} \quad (117)$$

This means that the efficiency of the motor is:

$$\eta_{motor} = \frac{P_0}{P_{inmotor}} = \frac{P_0}{P_0 + P_{losses}} \quad (118)$$

The PPU has an efficiency of η_{PPU} due to the switching of the switches, meaning that the total efficiency is:

$$\eta_{drive} = \eta_{motor} \eta_{PPU} \quad (119)$$

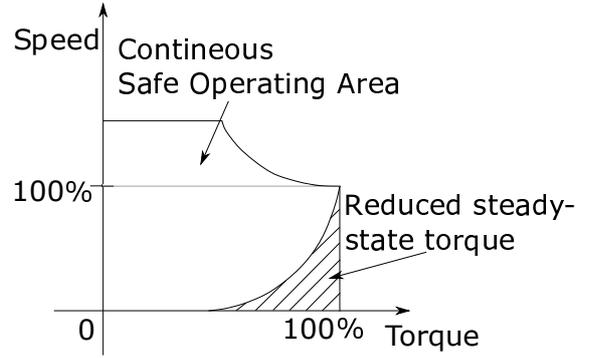


Figure 27: The continuous safe operating area and reduced steady state torque of motors without forced cooling

The efficiencies are often shown in the data sheets. In general is the total efficiency of small to medium motors 85% to 93%. The efficiency of PPU is normally between 93% and 97%. This means that the total efficiency of the drive is between 79% and 90%

2.4 Machine ratings

Note that electric motor ratings can be specified in several duty cycles, going from continuous operation to intermittent or dynamic duty cycles. The IEC duty cycles will be explained in [4].

Main issue of using an electric motor above its power rating, is the temperature rise in the motor. As a general rule of thumb, operating a motor $10^\circ C$ above its limit reduces the life expectancy by 50%.

The rated power often also contains the rated torque and rotational speed:

$$P_{rated} = \omega_{rated} T_{rated} \quad (120)$$

The operating regions are shown in Figure 27 for a drive without forced cooling. The graph shows that at a reduced torque, the motor can actually run at a higher speed. Note that the maximum speed is often limited by the bearings of the motor.

Secondly, there is a region at high torque but a rotational speed of less than ω_{rated} , where the motor cannot be run continuously without damage due to too high temperature in the motor. The motor heats up as the lower rotational speed reduces cooling, as normally the cooling fan is mounted on the main shaft of the motor. This means that in this region it is possible to operate, as long as the motor will also be given time to cool down again, meaning intermittent operation.

3 AC to DC Converters

There are several converters from AC to DC and vice versa. Often these use magnetic fields, often by using transformers, but not all. One example of an DC to AC converter that does not use magnetism, is the pulse-width modulation as shown in [3].

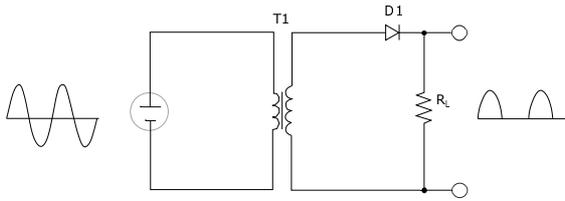


Figure 28: Schematic of a half-wave rectifier

DC to AC converters, also called rectifiers, will not have a steady direct current. It will always be in the same direction, so it is not alternating, but the voltage will fluctuate, also called 'pulsating DC'. As most DC components cannot handle this, filtering has to be applied after the rectifier to flatten the pulsations (inductors and/or capacitors).

3.1 Half-wave rectifier

The simplest rectifier is a half-wave rectifier, which is shown in Figure 28. Note that the resistor R_L is not part of the rectifier, but represents the load in the DC system. The half-wave rectifier uses a single diode $D1$ to allow for the positive half of the current (current in one direction), but to reject the current in opposite direction. In order to reject the positive side instead of negative, the diode is placed on the other side of the transformer.

For an ideal diode, the output voltage amplitude is the same as the input. Unfortunately, real diodes are not ideal, meaning that these have a threshold voltage before allowing the current through. This means that the amplitude of the output voltage amplitude is the amplitude at the input minus the threshold voltage. This means that not half of the AC is forwarded, but slightly less. This means that for efficiency the input amplitude should be much larger than the threshold voltage.

This shows that the half-wave rectifier is simple, but not very efficient.

3.2 Full-wave rectifier

A full wave rectifier uses two diodes $D1$ and $D2$, as shown in Figure 29, and enables to pass the positive as well as negative part of the AC voltage. Note that the negative port is connected to a center-tapped transformer as a common ground, to allow for current to flow. The frequency of the pulsating DC is twice that of the input AC frequency.

Note that in reality the diodes are not ideal, resulting in small time periods of 0V in the DC voltage, as the input

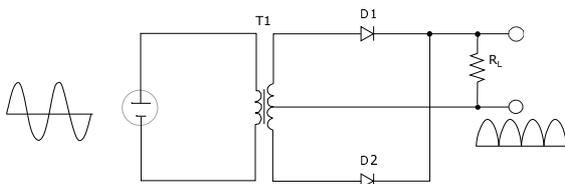


Figure 29: Schematic of a full-wave rectifier

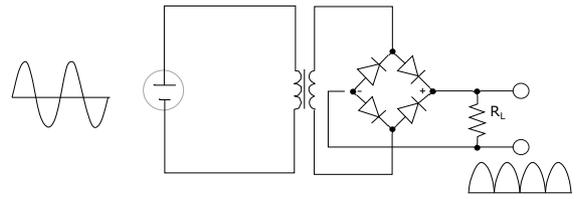


Figure 30: Schematic of a bridge rectifier

voltage is not above the threshold value.

3.3 Bridge rectifier

To prevent the need of a center-tapped transformer, a bridge rectifier can be used. It uses four diodes, as shown in Figure 30. Besides the advantage of the omitted center-tapped transformer, is that these are also commercial available as Integrated Circuit (IC). It then has only four connection pins, 2 for AC supply and 2 for DC output.

4 Ferrite Bead

Magnetism can interfere with conductors and induce a voltage, resulting in disturbances in the signals going through the cable. For sensor cables this was already discussed in Section 1.5. Ferrite beads can filter high frequency noise from a cable over a broad frequency range. Improper use however can lead to undesired resonance behaviour, for instance when used in combination with a decoupling capacitor used for low-pass filtering.

Ferrite beads are a passive component that prevents electromagnetic interference in two directions: From a device or to a device. The main way these work is described by Faraday's law: The magnetic core around a current-carrying conductor induces a back-emf for fast changing (and thus high frequency) signals. Ferrite is a magnetic material and thus when placed around a power supply or ground signal, it provides a source of inductive impedance for signals in the cable, but these are more complex than just impedance.

The ferrite beads are specified for certain voltage and current ratings. Exceeding the current rating can lead to damage of the component, mainly due to too much heat accumulation (see the hysteresis in Figure 3). A temperature increase can also lower the current rating drastically. When exceeding the DC current, the ferrite bead will 'saturate', which is non-linear behaviour and will look like lowering the (normally constant) inductance. .

4.1 Modeling of Ferrite bead

The ferrite bead uses magnetism to interfere with the signal in the cable. When looking at the formulas, the length l_m and the relative permeability μ_r can be varied, as well as the area A_m (assuming $n = 1$, as the bead is around one

wire):

$$B_m = \mu_m \frac{i}{l_m} = \frac{\mu_r}{\mu_0} \frac{i}{l_m} \quad (121)$$

$$\Phi = B_m A_m = \frac{\mu_r}{\mu_0} \frac{i}{l_m} A_m \quad (122)$$

Note that for the different types of ferrite, the relative permeability μ_r is between 10 and 20 000.

The models of the ferrite beads are however mostly electrical, as their effect on the electrical circuit is desired to be modeled, not the magnetism itself. The model is shown in Figure 32 and consists of a capacitor, inductor and a resistor in parallel, with a resistor in series. The resistor R_s is the DC resistance of the bead. The capacitance C is the parasitic capacitance. L is the bead inductance and R is the AC resistance.

Using this model, the size of the impedance $|Z|$ can be determined (see section 2.2 of [5]:

$$|Z| = \left| R_s + \frac{\frac{1}{j\omega C} j\omega L R}{-j\frac{1}{\omega C} + j\omega L + R} \right| \quad (123)$$

$$= \left| R_s + \frac{\frac{RL}{C}}{j\left(\frac{-1}{\omega C} + \omega L\right) + R} \right| \quad (124)$$

Now a sensitivity study can be done, using $R_s = 300m\Omega$, $R = 1.082k\Omega$, $L = 1.208\mu H$ and $C = 1.678pF$. The result is shown in 33. Using this impedance, the result on the electrical system can be estimated.

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- [4] J.G. Gruijters. Electric motors. *Not published*, 2021.



Figure 31: Practical examples of ferrite beads

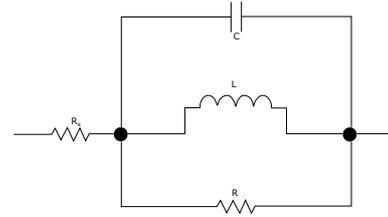


Figure 32: An electrical model of the ferrite bead

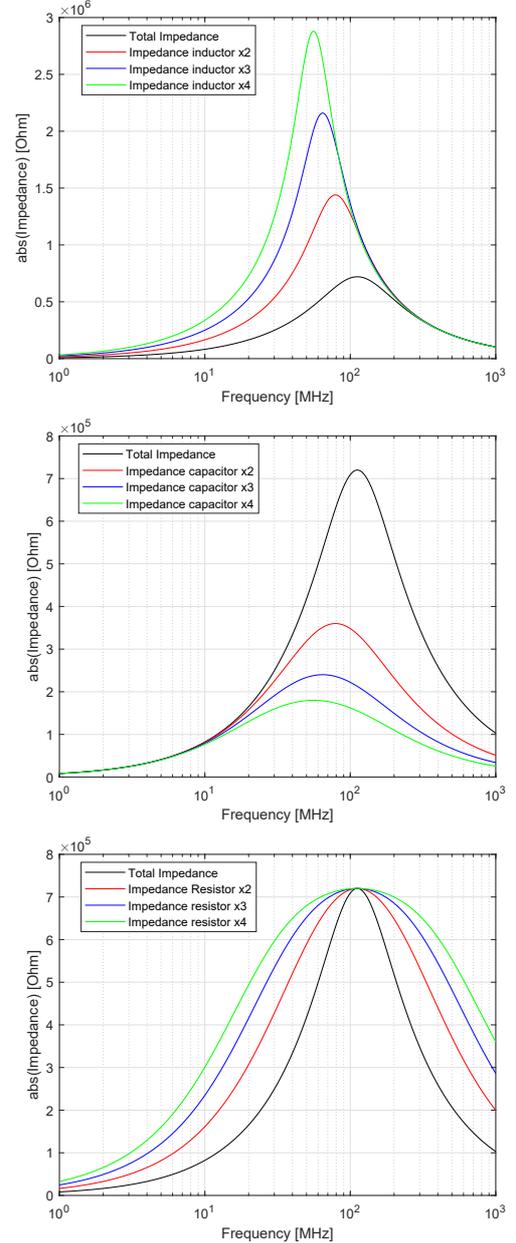


Figure 33: The electrical impedance of the ferrite bead, with changing inductance (top), capacitance (middle) and AC resistance (bottom)

- [5] J.G. Gruijters. Electrical analogy for mechanical engineers. *Not published*, 2021.
- [6] Ned Mohan. *Electric Drives*. MNPERE, 2003.