Electrical Analogy for Mechanical Engineers

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Abstract

Many mechanical engineers struggle with understanding electric drives and their basic working principle. Therefore, this article aims at showing the basic principles and show the mechanical analogues.

This overview is made while studying electrical drives and should be seen as notes taken during this study.

In the history, there was no distinction between the several fields of science. As more was known and the research was more into the details, the several fields within science are distinguishable. This also means that as science has progressed over time, the gap between mechanical engineering and electrical engineering grew.

Due to the gap in knowledge, the misunderstanding between mechanical and electrical engineers also grew. And although an electric drive will drive a mechanical system, the mechanical engineers are no longer fully familiar with the state of the art electrical drives, although this is becoming more and more important to the integral design of machinery.

Therefore, this article aims at giving the mechanical engineer a better understanding of how electrical drives work and what their limitations are. This overview is based on the book of Ned Mohan [3] and was made while studying this book, to enlarge my own knowledge of electrical drives.

1 Mechanical approach of electric drives

To start close to the mechanical engineers knowledge, the electrical drive itself has mechanical components and, together with the load, it is a mechanical system. A simple overview of the mechanical system is shown in figure 1. The electric motor is represented as a mass driven by the torque T_{em}. The motor mass as a moment of inertia[1] of J_{em} and

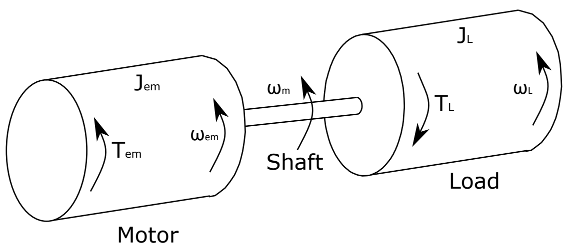

Figure 1: Simple schematic drawing of the mechanical overview of an electrical drive.

[1] All moments of inertia are here taken about the central axis of the motor, along which it is rotating.

a rotational speed ω_{em}. The motor is connected to the load through the shaft (gears and gear boxes are neglected in the first part). When the shaft is assumed to be infinitely stiff, the shaft rotational speed ω_m is equal to the motor speed. The load is again attached to the shaft, which means that for an infinitely stiff shaft the rotational speed of the load ω_L is equal to the rotational speed of the motor and the shaft. The rotational acceleration by a nett torque $T_J = T_{em} - T_L$ can be calculated by (in SI units):

$$T_J = J_{eq}\,\dot{\omega} \tag{1}$$

where $\dot{\omega}$ is the rotational acceleration of the motor as well as the load and shaft. The equivalent moment of inertia J_{eq} is calculated by taking the sum of the moments of inertia ($J_{eq} = J_{em} + J_L$).

The work done by the motor torque, is then:

$$dW = T_{em}\,d\theta \tag{2}$$

The required mechanical power of the electric drive is then:

$$p = T_{em}\,\omega_{em} = J_{eq}\frac{d\omega_{em}}{dt}\,\omega_{em} \tag{3}$$

Starting at $t = 0$ with no speed and no work done, the amount of energy required is then:

$$\begin{aligned} W = E &= \int_0^t p\,d\tau \\ &= J_{eq}\int_0^t \frac{d\omega_{em}}{d\tau}\,\omega_{em}\,d\tau \\ &= J_{eq}\int_0^{\omega_{em}} \omega_{em}\,d\omega_{em} \\ &= \frac{1}{2}\,J_{eq}\,\omega_{em}^2 \end{aligned} \tag{4}$$

This result is not surprising, as this is the general equation for rotational energy. The proof of the calculation is shown completely at [4].

1.1 Resonances

Only a small part of the book shows mechanical resonances, so this is made more comprehensive in this paper. This paper will show rotational resonances only, translational resonances are already shown in [2]. For a more comprehensive reading, please see [2].

For resonances, it is required that the shaft is no longer seen as infinitely stiff, but acts as a rotational spring between the motor and the load. The general equation of motion is:

$$J\,\frac{d^2\ddot{\theta}}{dt^2} + B\,\frac{d\dot{\theta}}{dt} + K\,\theta = T_{nett} \tag{5}$$

where T_{nett} is the nett torque on the mass. Now that the shaft has a limited stiffness, i.e. $K \neq \infty$, it means that the angles and rotational velocities of the motor and load are not equal by definition. The difference in angle for the undampened case will become:

$$\theta_{em} - \theta_L = \frac{T_{shaft}}{K} \tag{6}$$

When the shaft has a certain stiffness, it means that there are resonances in the system. This becomes abvious when the differential equation of 5 is solved analytically. This is done by taking the homogeneous differential equation (i.e. assume $T_{nett} = 0$ for now) and solving the homogeneous differential equation with the standard solution:

$$\begin{aligned} \theta &= e^{\lambda\,t} \\ \frac{d\ddot{\theta}}{dt} &= \lambda\,e^{\lambda\,t} \\ \frac{d^2\ddot{\theta}}{dt^2} &= \lambda^2\,e^{\lambda\,t} \end{aligned} \tag{7}$$

This will give the following equation:

$$J\,\lambda^2 + B\,\lambda + K = 0 \tag{8}$$

This second order equation is solvable with the quadratic formula and will deliver two solutions:

$$\lambda_1 = \frac{-B + \sqrt{B^2 - 4\,J\,K}}{2\,J} \tag{9}$$

$$\lambda_2 = \frac{-B - \sqrt{B^2 - 4\,J\,K}}{2\,J} \tag{10}$$

Using these two solutions, there are three possible outcomes:

- Underdamping: λ_1 and λ_2 are both complex numbers, as $B^2 < 4\,J\,K$. This leads to a general solution of $y = e^{\lambda\,t}\,(C_1\,cos(\omega\,t) + C_2\,sin(\omega\,t))$ for $\lambda = k \pm i\,\omega$.
- Critical damping: λ_1 and λ_2 are equal, $\lambda_{1,2} = \frac{-B}{2\,J}$ as $B^2 = 4\,J\,K$, which leads to the general solution of $y = (C_1 + C_2\,t)\,e^{\lambda\,t}$
- Overdamping: λ_1 and λ_2 are both real numbers, as $B^2 > 4\,J\,K$, which leads to the general solution of $y = C_1\,e^{\lambda_1\,t} + C_2\,e^{\lambda_2\,t}$

The frequency at which the resonance occurs, is for the undampened case:

$$\omega_n = \sqrt{\frac{K}{J}} \tag{11}$$

For the dampened case, the damped resonance frequency is:

$$\omega_n = \sqrt{\frac{K}{J} - \left(\frac{B}{2\,J}\right)^2} \tag{12}$$

In general (rule of thumb) the highest frequency of the system should at least 5 times lower than the resonance frequency in order to be able to neglect the resonances of a system.

1.2 Coupling mechanisms

As shown in the previous paragraph, it is good to have a high stiffness between the motor and the load, in order to suppress resonances and achieve high (and not reachable) resonance frequencies. In practice, coupling mechanisms are often used due to the following reasons:

- The rotational movement is converted to a linear mechanical movement;
- The electrical motors are often designed for high rotational speeds, but the mechanical motion needs to be much lower;
- The axis of rotation is changed.

The introduction of coupling mechanisms come however at a price. The coupling mechanisms result in power losses in the system, can lead to non-linearities in the system and are subject to wear and tear.

One of the most common mechanisms are gears as used in gearboxes. In general the speed of the gears and power are conserved, thus:

$$r_1\,\omega_1 = r_2\,\omega_2 \tag{13}$$

$$T_1\,\omega_1 = T_2\,\omega_2 \tag{14}$$

Then the load and electrical drive torque and inertia are substituted for the torques:

$$T_1\,\frac{\omega_1}{\omega_2} = T_2 \tag{15}$$

$$(T_{em} + J_{em}\,\dot{\omega_{em}})\,\frac{\omega_{em}}{\omega_L} = T_L + J_L\,\dot{\omega_L} \tag{16}$$

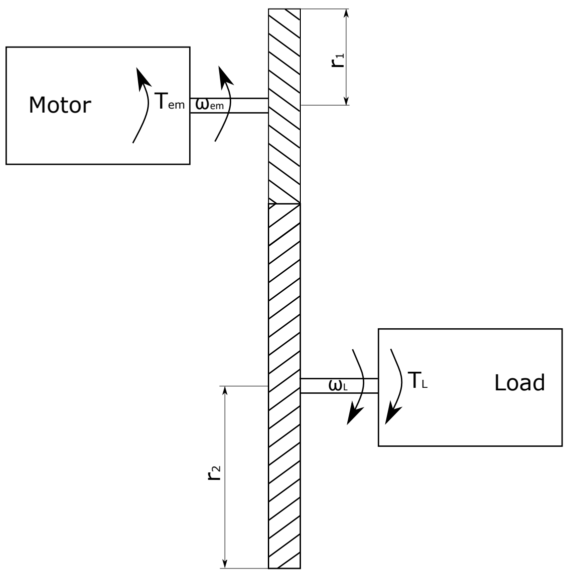

Figure 2: Sketch of gears between the motor and the load.

The equivalent moment of inertia is now:

$$
\begin{aligned}
J_{eq} &= J_{em} + \left(\frac{\omega_L}{\omega_{em}}\right)^2 J_L \qquad (17)\\
&= J_{em} + \left(\frac{r_1}{r_2}\right)^2 J_L
\end{aligned}
$$

A direct conclusion is that when $\frac{r_1}{r_2} < 1$, the load moment of inertia becomes less important to the required motor torque. The required motor torque is now (see for an example Figure 3):

$$
\dot{\omega_L} = \frac{d\omega_L}{dt} = \frac{d\omega_{em}}{dt}\frac{\omega_L}{\omega_{em}} \qquad (18)
$$

$$
\begin{aligned}
T_{em} &= \left(J_{em} + \left(\frac{\omega_L}{\omega_{em}}\right)^2 J_L\right)\frac{d\omega_{em}}{dt} + \frac{\omega_L}{\omega_{em}} T_L\\
&= J_{eq}\frac{d\omega_{em}}{dt} + \frac{\omega_L}{\omega_{em}} T_L \qquad (19)
\end{aligned}
$$

For an optimal result, and thus a small as possible electric drive, the motor torque needs to be minimised. The engineer can choose the gear ratio. For now assume that the load torque is negligible ($T_L = 0$). The motor acceleration $\frac{d\omega_{em}}{dt}$ still scales with the gear ratio and is thus also a function of the gear ratio:

$$
\frac{d\omega_{em}}{dt} = \frac{d\omega_L}{dt}\frac{\omega_{em}}{\omega_L} \qquad (20)
$$

This means that the motor torque in equation 19 can be minimised by taking:

$$
\begin{aligned}
J_{em} &= \left(\frac{r_1}{r_2}\right)^2 J_L\\
\left(\frac{r_1}{r_2}\right)^2 &= \frac{J_{em}}{J_L} \qquad (21)
\end{aligned}
$$

When substituting this in equivalent moment of inertia equation, the optimum is found when the equivalent moment of inertia is equal to twice the motor inertia ($J_{eq} = 2J_{em}$). This means that the load inertia felt by the electrical drive should be the same as the motor moment of inertia, i.e. $\left(\frac{r_1}{r_2}\right)^2 J_L = J_{em}$.

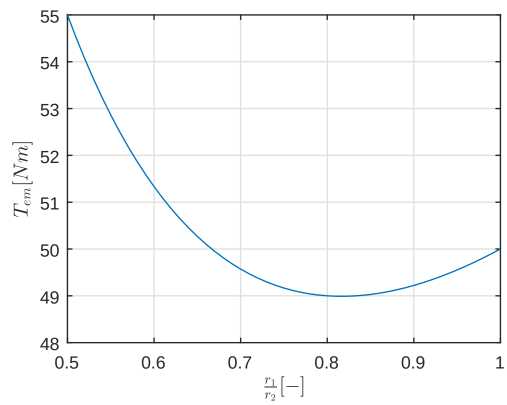

Figure 3: Plot of the required motor torque for the example of $J_{em} = 10kg\,m^2$, $J_L = 15kg\,m^2$, $T_L = 0$ and a load acceleration of $2\frac{rad}{s^2}$.

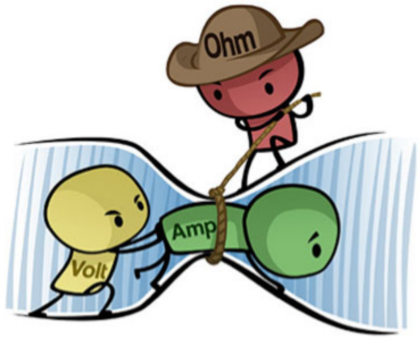

Figure 4: Simple explanation of the electrical quantities electrical potential (Volt), current (Ampere) and resistance (Ohm).

1.3 Electrical analogy

To familiarize the mechanical engineer with the electrical systems, it is possible to use electrical analogy for mechanical quantities. This makes it easier to remember the relationships. In table 1 the analogy is shown. In Figure 4 a simplified representation is shown for the three main electrical quantities (inductance is not shown).

Now lets have one small example, just to show the analogy. For simplicity, take the motor-load combination with an infinitely stiff shaft as shown in figure 1. The torque of

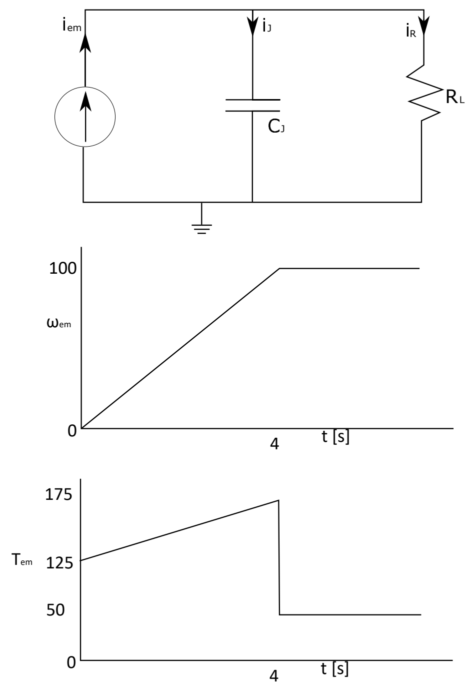

Figure 5: Plot of the example for the electrical analogy.

Table 1: The electrical analogy for mechanical quantities

Mechanical	**Electrical**
Torque T	Current i
Angular speed ω	Voltage v
Angular displacement θ	Flux linkage ψ
Moment of inertia J	Capacitance C
Sping constant K	1/inductance $\frac{1}{L}$
damping coefficient B	1/Resistance $\frac{1}{r}$
Coupling ratio $\frac{n_{em}}{n_L}$	Transformer ratio $\frac{n_L}{n_{em}}$

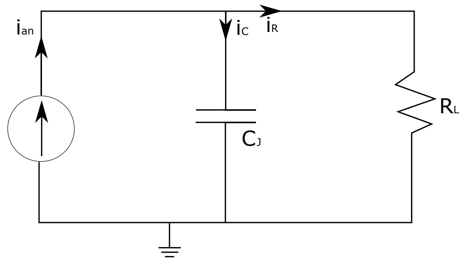

Figure 7: A basic RC electric circuit with components parallel as example.

the load T_L has viscous friction and will rise linearly according $T_L = B\,\omega_L = 0.5\,10^{-3}\,\omega_L\ [Nm]$. The combined inertia is $J_{eq} = 5\,10^{-3}\ [kg\,m^2]$. The rotational speed increases linearly over 4 seconds from 0 to 100 $\frac{rad}{s}$. Calculate the electromagnetic torque required for this action.

The electrical analogy is drawn in figure 5. To start with the capacitance, the $C_J = J_{eq} = 5\ [mF]$. The resistor is equal to the damping as shown in table 1. The damping is shown in this case by the viscous friction of T_L, and is thus $R = \frac{1}{B} = \frac{1}{0.5\,10^{-3}} = 2000[\Omega]$. The linear acceleration has an analogy with the change of voltage in the system, thus $\frac{dv}{dt} = \frac{d\omega_{em}}{dt} = 25\ \left[\frac{V}{s}\right]$.

In analogy with the mechanical equation $T_J = J_{eq}\,\dot{\omega}$, the current to the capacitor is $i_J = C\,\frac{dv}{dt} = 125\ [mA]$. The current through the resistor has an analogy with the friction of the load. The friction is $T_L = B\omega_L$ and thus is the current to the resistor $i_R = \frac{v(t)}{R} = \frac{25\,t}{2000} = 12.5\,t[mA]$.

Between 0 and 4 seconds, the total current / torque is $T_{em} = (125 + 12.5\,t)\,10^{-3}[N\,m]$.

After 4 seconds only the viscous friction is present, which leads to $T_{em} = 12.5\,4\,10^{-3} = 50\,10^{-3}[Nm]$.

2 Basics of electric circuits

In this paragraph, small letters (v, i) will be used for instantaneous values. Maximum (amplitude) values are shown as $\hat{V}$.

For electric circuits, it is possible to create differential equations similar as done for mechanics. Kirchhoff's laws help to achieve this. For components in series, Kirchhoff's voltage low is applicable. An example is shown in figure 6 with an inductor (or coil), resistor and capacitor. The voltage law of Kirchhoff states that ***the directed sum of the electrical potential differences (i.e. voltage) around any closed circuit is zero***. In mathematics this means $\sum_{k=1}^{n} v_k = 0$. The voltage over the three components is:

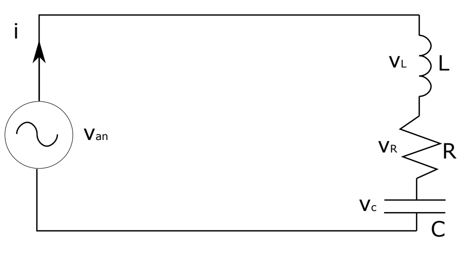

Figure 6: A basic RLC electric circuit with components in series as example.

$$v_L = L\,\frac{di(t)}{dt} \tag{22}$$

$$v_R = R\,i(t) \tag{23}$$

$$v_C = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + V_C(0) \tag{24}$$

$$\stackrel{V_C(0)=0}{\Longrightarrow} v_C = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt$$

The differential equation for the circuit in figure 6 will be (the source has a voltage of V_{an}):

$$v_{an} + (-v_L) + (-v_R) + (-v_C) = 0 \tag{25}$$

$$v_{an} = L\,\frac{di(t)}{dt} + R\,i(t) + \frac{1}{C}\int_{t_0}^{t} i(t)\,dt \tag{26}$$

For parallel systems, as for example in Figure 7, Kichhoff's current law is applicable, which is also known as the junction, nodal or point rule. This rule states that ***in any node in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of the node***. In mathematical way this is $\sum_{k=1}^{n} i_k = 0$.

By rewriting equations 22 till equation 24, the differential equation would become:

$$i_{an} + (-i_R) + (-i_C) = 0 \tag{27}$$

$$i_{an} = \frac{v_R}{R} + C\,\frac{dv_C}{dt} \tag{28}$$

2.1 Phasor representation

For electric circuits there is a more simplistic way to calculate the responses of a circuit using phasors. A phasor is a complex vector which represents a sinusoidal function. The sinusoidal function can be written as a complex number by using Eulers formula, as shown in figure 8. This means that the voltage sinusoidal signal can be rewritten as:

$$v(t) = \hat{V}\,cos(\omega\,t) = \hat{V}\,\frac{e^{j\,(\omega\,t-\varphi)} + e^{-j\,(\omega\,t-\varphi)}}{2} \tag{29}$$

$$= Re\left(\hat{V}\,e^{j\,(\omega\,t-\varphi)}\right) \tag{30}$$

The phasor is the complex number $\hat{V}\, e^{j\,(\omega\, t - \varphi)}$ in this case. The phasor is denoted with a bar on top of the variable, i.e. $\bar{V} = \hat{V}\, e^{j\,(\omega\, t-\varphi)}$. Another (shorter) method to note the phasor, is by only using the polar vector notation $\bar{V} = \hat{V}\angle - \varphi$.

Using the phasor, the quantities with different phases are easily added together by adding the complex numbers together. This can be done graphically in the complex plane or by adding the real and imaginairy parts of the complex numbers.

2.2 Impedance and Admittance

Impedance Z is the ratio of the voltage phasor to the electric current phasor. This ratio gives a measure for the opposition to varying electric current in the system. This way, impedance is a measure for the resistance in AC circuits, as it possesses both a magnitude and a phase, unlike resistance[2]. The impedance Z is defined as:

$$\bar{I} = \frac{\bar{V}}{Z} \tag{31}$$

The impedance is frequency dependent, as the magnitude of the several phasors change with frequency. For the three components as shown in figure 6 and 7, the impedance is calculated as:

$$Z_L = j\, X_L = j\, \omega\, L \tag{32}$$
$$Z_R = R \tag{33}$$
$$Z_C = -j\, X_C = -j\, \frac{1}{\omega\, C} \tag{34}$$

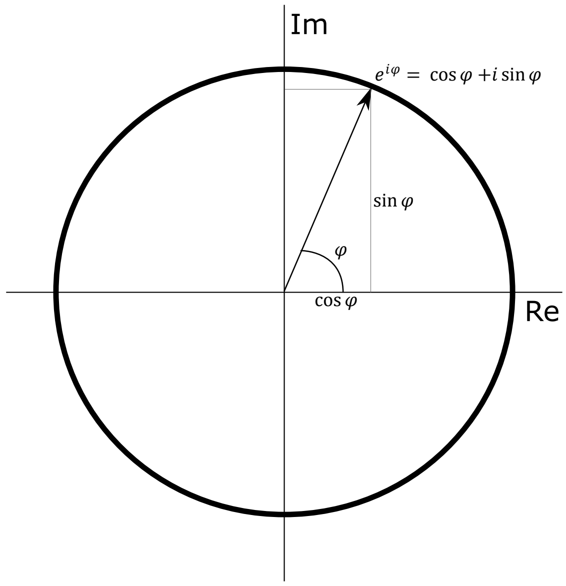

Figure 8: The complex plane with a phasor of length 1 and an angle φ. Euler's formula for complex numbers is shown as well in this picture.

[2]Resistance is often used in DC circuits, but is not suitable for AC circuits as it lacks the phase. In DC circuits there is no difference between impedance and resistance.

As the impedance can also be expressed in a magnitude and phase. For the example of figure 6 it is:

$$Z = |Z|\angle\varphi \tag{35}$$
$$|Z| = \sqrt{R^2 + \left(\omega\, L - \frac{1}{\omega\, C}\right)^2} \tag{36}$$
$$\varphi = tan^{-1}\left(\frac{\omega\, L - \frac{1}{\omega\, C}}{R}\right) \tag{37}$$

Important to note is that the impedance Z does not have a bar on top. Although the impedance is a complex quantity, it is ***not*** a phasor and does ***not*** have a time domain expression.

The impedance is calculated by adding the impedance of the components which are in series. For components in parallel, the impedance of the components are multiplied, which is then divided by the sum of the impedance of the components. This means that for figure 7 the impedance is:

$$Z = \frac{\left(-j\frac{1}{\omega\, C}\right)\, (R)}{\left(-j\frac{1}{\omega\, C}\right) + (R)} \tag{38}$$

The inverse of impedance is called admittance Y.

$$Y = \frac{1}{Z} \tag{39}$$

It is important to notice that the calculation of the impedance and the current phasor using equation 31 is much easier than solving the differential equations as shown in equation 26.

2.3 Power

In figure 10 two sub-circuits are drawn. Each sub-circuit can contain passive (R-L-C) elements as well as active elements. In figure 10 the current $i(t)$ is drawn positive from the + pole of sub-circuit 1 to the + pole of sub-circuit 2. The

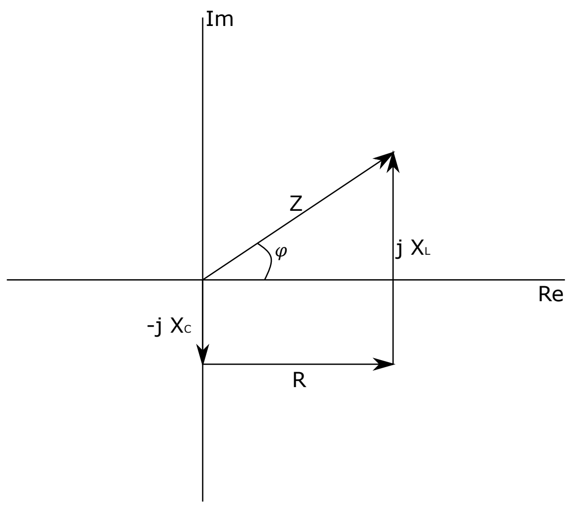

Figure 9: The impedance of figure 6 shown in the complex plane.

current is always positive from the + pole to the − pole. Electrons on the other hand, have a negative charge and flow from the − pole to the + pole. This means that the power is flowing from the − pole of sub-circuit 1 to the − pole sub-circuit 2. Again, positive power is defined in figure 10 as flowing from sub-circuit 1 to sub-circuit 2.

The power is the product of the current and voltage, which means that the instantaneous power is:

$$p(t) = v(t)\, i(t) \tag{40}$$

When the current and voltage have exactly the same

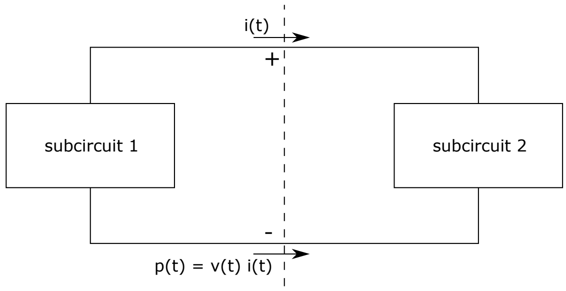

Figure 10: Simple overview of two subcircuits.

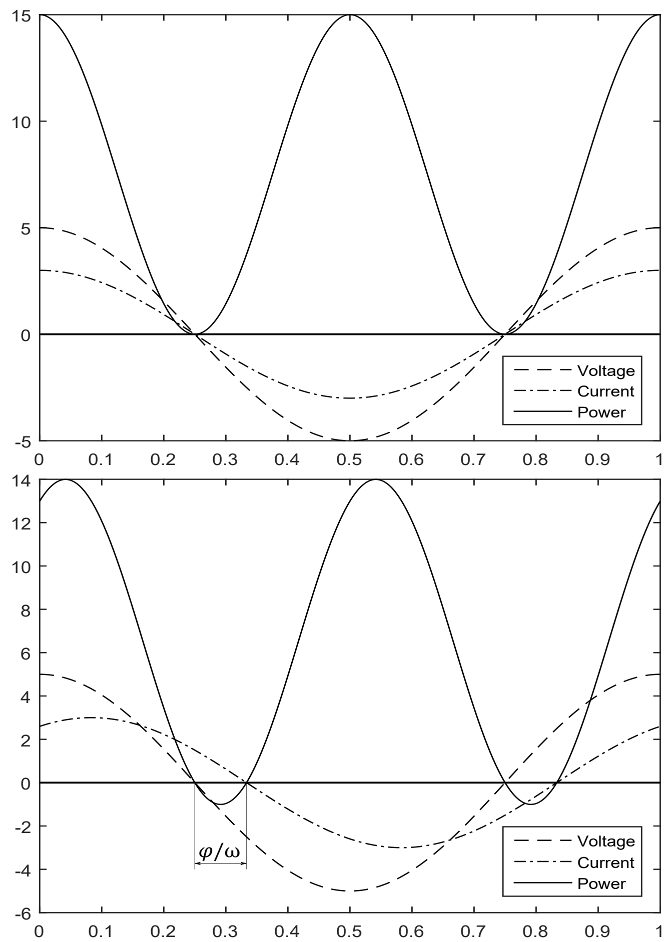

Figure 11: Result in power for a system without and with a phase shift.

phase, as shown in the top plot of figure 11, the power is always positive ($p(t) \geq 0$). This means that the power always flows from sub-circuit 1 to sub-circuit 2.

Now assume that the current $i(t)$ lags behind with respect to the voltage, as shown in the bottom plot of figure 11. This means that $p(t)$ becomes negative during each half period, meaning that the instantaneous power is negative, and thus flows in opposite direction, during the time interval of $\frac{\varphi}{\omega}$ seconds per half cycle. This means that the real (average) power is not optimally transferred from one sub-circuit to the other.

To examine more closely what happens, the current and voltage phasors are used:

$$\bar{I} = \hat{I}\angle\varphi_i \tag{41}$$

$$\bar{V} = \hat{V}\angle\varphi_v \tag{42}$$

In the complex plane, complex numbers have a complex conjugate. This complex conjugate has the same magnitude, but the imaginairy part is multiplied by -1, which means that it is mirrored with respect to the real axis. The complex conjugate is denoted by a $*$:

$$\bar{I}^* = \hat{I}\angle-\varphi_i \tag{43}$$

$$\bar{V}^* = \hat{V}\angle-\varphi_v \tag{44}$$

An example of the complex conjugate $\bar{I}^*$ is shown in the left plot of figure 12.

To examine the average power supplied by sub-circuit 1 to sub-circuit 2, as shown in the beginning of this paragraph, the complex power S is used, which uses the complex conjugate of the current:

$$S = \frac{1}{2}\,\bar{V}\,\bar{I}^* \tag{45}$$

Up till now, the phasors used the amplitude. In electric engineering, it is more convenient to calculate the real, reactive and complex powers using the Root Mean Square (RMS) value for the magnitude. For the sinusoidal functions $\bar{I}$ and $\bar{V}$ this means:

$$I = \frac{\hat{I}}{\sqrt{2}} \tag{46}$$

$$V = \frac{\hat{V}}{\sqrt{2}} \tag{47}$$

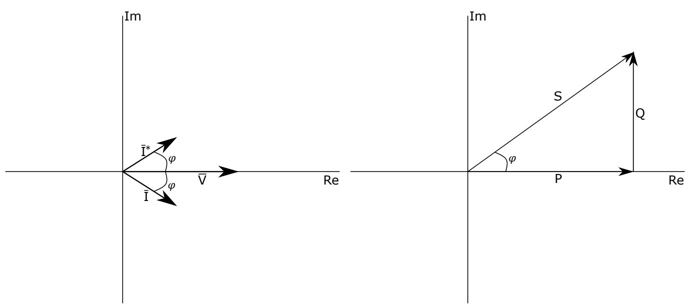

Figure 12: The current, phase and complex power drawn in the complex plane.

The complex (also called the apparent) power can now be simplified to:

$$S = V\angle\varphi_v \, I\angle - \varphi_i = V \, I \, \angle(\varphi) \tag{48}$$

where $\varphi = \varphi_v - \varphi_i$. An example of the complex power is shown in the right plot of figure 12. The complex power is the power which is needed from sub-circuit 1 to achieve the real power P at sub-circuit 2. This means that this complex power includes the inefficiency of the circuit due to the so-called $cos(\varphi)$ shift, as shown in figure 11 in the bottom graph. The unit of the complex power is $[V \, A]$.

The real part of the complex power is the real power P in $[W]$. The real power for the simple system as shown above is calculated using:

$$P = V \, I \, cos(\varphi) \tag{49}$$

The real power represents the real work done, in this case in sub-circuit 2.

The imaginairy part of the complex power is called the reactive power Q in $[VAR]$ (Volts-Ampere-Reactive). The reactive power is calculated using:

$$Q = V \, I \, sin(\varphi) \tag{50}$$

The reactive power is in this case the inefficient power due to the phase shift of the current. In most cases this reactive power is desired to be made as small as possible.

Using the definitions above, the following relations can be used:

$$\varphi = \varphi_v - \varphi_i \tag{51}$$

$$S = V \, I \, \angle(\varphi) = P + j \, Q \tag{52}$$

$$|S| = \sqrt{P^2 + Q^2} \tag{53}$$

$$\varphi = tan^{-1}\left(\frac{Q}{P}\right) \tag{54}$$

To express how efficient a circuit is, the power factor is defined:

$$powerfactor = \frac{P}{|S|} = \frac{P}{V \, I} = cos(\varphi) \tag{55}$$

This shows that the phase shift $cos(\varphi)$ between the current and voltage is directly related to the efficiency.

2.4 Three phase systems

Almost all electrical drives above a few kW use three phase AC circuits. The main advantage of 3 phase electrical AC induction motors is that these are simple, low-cost and low maintenance. The induction motor has a stator and a rotor. The stator has the three 'spools'[3] which are fed by three AC currents, but shifted 120° with respect to each other. This creates a rotating magnetic field (rotating due to the AC current). By electromagnetic induction the rotor bars will have a current (this is transformer action). Now that there is a current in a magnetic field and Lawrence law dictates that this results in a force acting on the rotor, which makes it spin in the same direction as the rotating magnetic field. See [1] for more information.

At the start of the electric motor, the rotor is standing still while the rotating magnetic field is rotating. This means that the magnetic field crosses the rotor bars quickly. This results in a huge current in the rotor bars, which needs to be powered by the current through stator coils. This draws a huge current, which can result in a voltage drop in the power line.

Older non-adjustable AC circuits use 'wye' (also called star) and 'delta' connections for starting to prevent the huge current demand and resulting voltage drop in the power line. The wye and delta connections are shown in figure 13.

2.4.1 Star or wye connection

In a balanced condition, meaning that the three voltages over the coils are equal in magnitude but shifted 120°, and under sinusoidal steady state conditions, it is known that:

$$\bar{V}_{an} = \hat{V}_s \angle 0° \quad \bar{V}_{bn} = \hat{V}_s \angle - 120° \quad \bar{V}_{cn} = \hat{V}_s \angle - 240° \tag{56}$$

where $\hat{V}_s$ is the voltage amplitude. As the phase shift is 120°, it is known that:

$$\bar{V}_{an} + \bar{V}_{bn} + \bar{V}_{cn} = 0 \tag{57}$$

As all three lines are coupled to a neutral line instead of to each other, the voltage over the coils in the motor are equal to the voltage of the supply $\bar{V}_{an}$, $\bar{V}_{bn}$ and $\bar{V}_{cn}$.

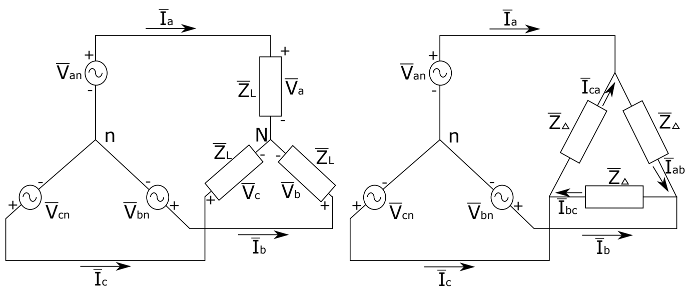

Figure 13: On the left the star (wye) configuration is shown. On the right the delta configuration is shown.

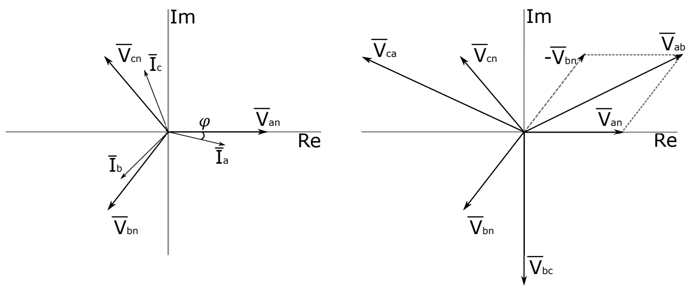

Figure 14: The voltage and current phasor diagrams for a three phase for a balanced system with sinusoidal steady state.

[3] This statement of three 'spools' is true for a two pole motor. Other motors can have more spools, which delivers more poles. For clarity the two pole motor is used here for illustration.

When a system is balanced and the impedance of the coils are equal, a motor can be analysed by looking at one phase, as the others will be the same. When equation 35 is used, the current can be calculated:

$$\bar{I}_a = \frac{\bar{V}_{an}}{Z_L} = \frac{\hat{V}_s}{|Z_L|}\angle - \varphi \tag{58}$$

$$\bar{I}_b = \frac{\bar{V}_{bn}}{Z_L} = \frac{\hat{V}_s}{|Z_L|}\angle - \frac{2\,\pi}{3} - \varphi \tag{59}$$

$$\bar{I}_c = \frac{\bar{V}_{cn}}{Z_L} = \frac{\hat{V}_s}{|Z_L|}\angle - \frac{4\,\pi}{3} - \varphi \tag{60}$$

This shows the current for the wye connection. This is also shown in the phasor graph on the left of figure 14.

2.4.2 Delta connection

For the delta connection the voltage over the spools is not equal to the voltage over the supplies, as the coils are connected to a supply at either side. This means that the coils have a voltage $\bar{V}_{ab}$, $\bar{V}_{bc}$ and $\bar{V}_{ca}$:

$$\bar{V}_{ab} = \bar{V}_{an} - \bar{V}_{bn} = \sqrt{3}\hat{V}_s\,\angle 30^\circ \tag{61}$$

$$\bar{V}_{bc} = \bar{V}_{bn} - \bar{V}_{cn} = \sqrt{3}\hat{V}_s\,\angle - 90^\circ \tag{62}$$

$$\bar{V}_{ca} = \bar{V}_{cn} - \bar{V}_{an} = \sqrt{3}\hat{V}_s\,\angle - 210^\circ \tag{63}$$

The voltages $\bar{V}_{ab}$, $\bar{V}_{bc}$ and $\bar{V}_{ca}$ are also shown on the right in figure 14.

The current is therefore:

$$\bar{I}_a = \frac{\bar{V}_{ab}}{Z_\Delta} = \sqrt{3}\frac{\hat{V}_s}{|Z_\mathbf{L}|}\angle 30^\circ - \varphi \tag{64}$$

$$\bar{I}_b = \frac{\bar{V}_{bc}}{Z_\Delta} = \sqrt{3}\frac{\hat{V}_s}{|Z_\mathbf{L}|}\angle - 90^\circ - \frac{2\,\pi}{3} - \varphi \tag{65}$$

$$\bar{I}_c = \frac{\bar{V}_{ca}}{Z_\Delta} = \sqrt{3}\frac{\hat{V}_s}{|Z_\mathbf{L}|}\angle - 210^\circ - \frac{4\,\pi}{3} - \varphi \tag{66}$$

Another approach for the balanced condition with sinusoidal steady state is to use the star equation, but translate the delta impedance to the star impedance:

$$Z_L = \frac{Z_\Delta}{3} \tag{67}$$

To prove that the root is not forgotten, Ohm's law and the complex power equation need to be combined. To start with Ohm's law:

$$|V| = |I|\,|Z| \tag{68}$$

$$|I| = \frac{|V|}{|Z|} \tag{69}$$

The complex power, as shown in equation 48, is:

$$|S| = |I|\,|V| \tag{70}$$

Now solve the complex power formula while substituting the current I:

$$|S| = \frac{|V|\,|V|}{|Z|} \tag{71}$$

$$|Z| = \frac{|V|^2}{|S|} \tag{72}$$

This shows that when $|V|$ becomes $\sqrt{3}$ times as large, the impedance needs to become 3 times as large due to the square.

2.4.3 Difference between star and delta

With the same power supply, the star connection draws much less current (factor $\sqrt{3}$). This means that the start-up with the star connection draws less high starting current due to the high difference in speed between the rotating magnetic field and the rotor.

For the delta connection, the current is larger through the coils, which means that the motor can deliver more torque. This means that when using the delta connection, the motor can deliver more power when using the same power supply.

2.5 DC motor

The motor discussed in the previous paragraph is the AC motor. The DC motor is not mentioned in [3], but is nice to show here as well. Figure 15 shows the basic construction of a brushed DC motor. The 'wire' is placed in a magnetic field. When a current runs through the wire, a force acts on the wire. The direction of the force can be shown by using Fleming's left-hand rule: take your left hand and let the thumb, index finger and middle finger make 90° angles. When the middle finger points in the direction of the current (from + to -) and index finger in the direction of the magnetic field (from north to south), then the thumb points in the direction of the electromagnetic force.

As the motor rotates, the brushes switch of polarity, which means that as soon as the current should change direction to keep the motor rotating in the same direction, it also changes direction. For more smooth operation and/or prevention of equilibrium positions (for instance on the switch over point), the motor can be made of several wire loops. The loops will take turns to act as a magnet and make sure the motor operates in all circumstances.

There are also brush-less DC motors, but those look a lot more like AC motors with a stator and a rotor. The rotor is now a permanent magnet. The stator has multiple coils. When a current runs through the coil, it works as an electrical magnet, creating a north and a south pole. The opposite poles attract, which makes the brush-less DC motor rotate. When the magnet is close to the actuated coil, the next coil

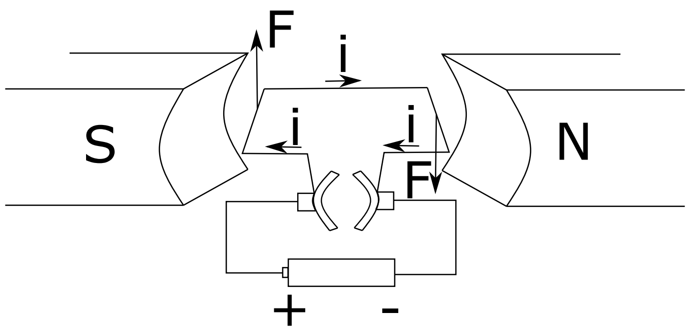

Figure 15: A schematic drawing of a brushed DC motor.

is activated while the other is deactivated. When the motor is rotated half a turn, the first coil has a current running in the opposite direction, to keep the motor running continuously. A way to get a larger power output, the coils are not only used to pull the magnet to the next coil, but it is also pushed away by the coil it just passed. Besides a more efficient motor, this will also make the torque output more stable. A brush-less motor does require however a controller to know when to energize the coils, and it requires a sensor to know the stator position.

3 Practice: Use of switches in proportional circuits

In electric circuits, especially sensor, measurement or proportional signal circuits (i.e. proportional valve signal), the choice of sensor / valve type also determines the electrical signal used to communicate the measured value to the PLC. There are roughly two commonly used types:

1. 4-20mA signal;
2. +/- 10V signal.

For redundancy reasons or due to splitting the the signal, splitters or switches are used. This means that there are small resistors in the circuit that can distort the proportional signal. To lower the resistance gold-plated contacts are used, but still this can lead to distortions, especially for +/-10V signals. This means that in that case the gold-plated contacts in combination with 4-20mA signals are preferred.

4 Practice: Peak current during start-up of the motor

In this article the impedance is shown. This is used when a motor starts up from 0rpm to its desired rotational speed. During the start-up, the motor will draw extra current from the power source. The power source and the circuit breaker must be rated for the start-up current, as otherwise the circuit breaker will trip before the motor gets up to speed.

In general it is therefore advantageous to lower the required power, and therefore the required current, as much as possible during start of the motor. This can for instance be done by disabling a pressure build-up on a pump connected to the motor, which will lower the required hydraulic power and consequently the electrical power.

When looking at the formulas, Equations 31 and 35 are here repeated. The impedance is based on Figure 6 without the capacitor (assuming that a motor consists mainly of coils L and a resistance R):

$$\bar{I} = \frac{\bar{V}}{Z} \tag{73}$$

$$Z = |Z|\angle\varphi \tag{74}$$

$$|Z| = \sqrt{R^2 + (\omega\, L)^2} \tag{75}$$

To simplify these expressions, the following is done:

$$V = |V|\, e^{j\,(\omega\, t+\phi_V)} \tag{76}$$

$$I = |I|\, e^{j\,(\omega\, t+\phi_I)} \tag{77}$$

$$Z = \frac{V}{I} = \frac{|V|}{|I|}\, e^{j\,(\phi_V-\phi_I)} \tag{78}$$

Now assume $\theta = \phi_V - \phi_I$:

$$Z = \frac{|V|}{|I|}\, e^{j\,\theta} \tag{79}$$

$$|Z| = \frac{|V|}{|I|} \tag{80}$$

$$\phi_V = \phi_I + \theta \tag{81}$$

$$\theta = arg(Z) \tag{82}$$

where $arg(Z)$ is the argument of the complex number of Z, meaning it is the angle of Z with the positive real axis in the complex plane.

From this it can be derived that:

$$|I| = \frac{|V|}{|Z|} = \frac{|V|}{\sqrt{R^2 + (\omega\, L)^2}} \tag{83}$$

Now with ω being 0rpm (or at least very small), the amplitude of the current becomes the largest (larger than for any other positive ω).

In concept stages of a project, a rule of thumb can be used to make a first estimation of the current peak required during starting:

- Direct online motor: Factor 8 of nominal current/power;
- Star - Delta motor: Factor 4 of nominal current/power;
- Softstarter: Factor 1.5 of nominal current/power;
- Frequency drive: Factor 1 of nominal current/power.

5 Practice: Automatic circuit breakers

Automatic circuit breakers are used in almost every circuit to prevent damage due to too high currents, for instance due to short circuits. The circuit breakers should however not to interfere with normal operation, also not during the peak currents as seen in the previous section.

In order to standardize the circuit breakers, different types are defined. Each type has a specific graph, as shown in Figure 16. The curve is valid for an ambient temperature of $30^o C$. Miniature circuit breakers have fixed trip settings, so changing the setting means replacing the circuit breaker. Larger circuit breakers can have adjustable trip settings. The setting mentioned on the circuit breaker is the rated current, which is the maximum of what the circuit breaker is designed to have continuously. The actual current at which it trips, is dependent on the type.

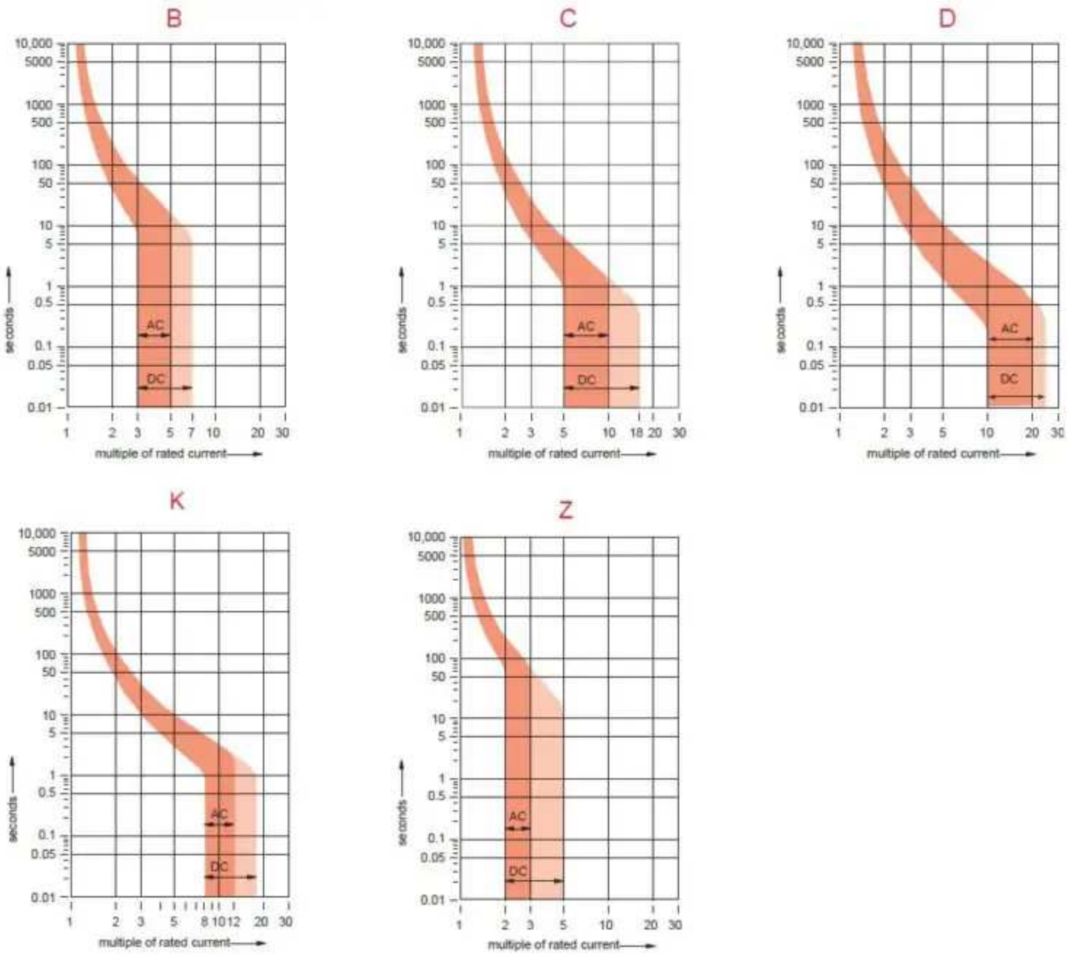

Figure 16: The different trip curves of the types of circuit breakers.

The circuit breaker has a magnetic and a thermal trigger, as shown in Figure 17. The thermal trigger means it will take time for the breaker to trip on that over-current. The magnetic trigger means the over-current is not required to be there for a longer period, although it is not instantaneous, as is shown in Figure 16. The area shown in Figure 16 is the area where the circuit breaker will trigger. To make absolutely sure the circuit breaker will trigger, the current must be on the right side of the area, not inside the area.

The circuit breakers are denoted with the type character and a number indicating the current setting (but without the A of Ampere), i.e. B5 means a B-type circuit breaker with a rated current of 5Ampere.

In the overview below, mainly the AC trip currents are shown. Note that the DC area for the trip is larger than for the AC current.

5.1 Type A miniature circuit breaker

The type A miniature circuit breaker (MCB) is a highly sensitive circuit breaker that trips instantaneously when the current reaches 2 to 3 times the rated current. This type is not very common to use, due to its sensitivity.

5.2 Type B miniature circuit breaker

The type B MCB has a magnetic trip at 3 to 5 times the rated current, when that current exceeds 0.01 to 10 seconds. This type is used for mainly resistive loads, meaning that there is no inductance to a very limited amount of inductance in the circuit. Main applications are therefore low power domestic applications, e.g. lighting circuits.

5.3 Type C miniature circuit breaker

The type C MCB has a magnetic trip at 5 to 10 times the rated current that exceeds 0.01 to 1 second. This allows for inductive loads in the circuit, like electric motors and transformers. This type is thus mainly used in commercial and industrial applications.

5.4 Type D miniature circuit breaker

The type D MCB has a magnetic trip at 10 to 20 times the rated current that exceeds 0.01 to 0.2 seconds. This type is used for high inductive loads, like heavy motors, transformers, X-ray machines and welding circuits. This type is thus often used in high power applications.

5.5 Type K miniature circuit breaker

The type K MCB has a magnetic trip at 8 to 12 times the rated current that exceeds 0.01 to 1 second. This type is commonly used for inductive loads that have a chance of high inrush currents.

5.6 Type Z miniature circuit breaker

The type Z MCB has a magnetic trip at 2 to 3 times the rated current that exceeds 0.01 to 50 second. This means the Z type MCB is between the A and B type and mainly used for non-inductive loads only.

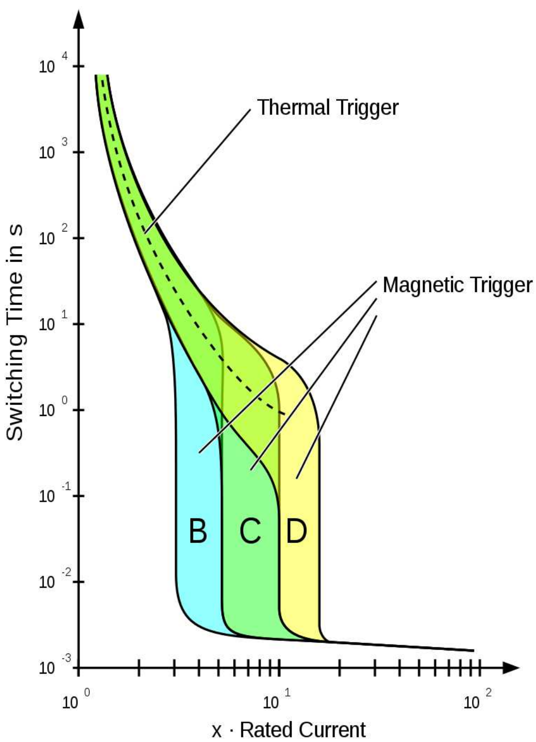

Figure 17: Explanation of the different regions (magnetic and thermal trigger) of the trip curve of the circuit breaker.

References

[1] Learn Engineering. How does an induction motor work? *INTERNET LINK*, -.

[2] J.G. Gruijters. Basics of linear control theory for mechanical engineering. *Not published*, 2018.

[3] Ned Mohan. *Electric Drives*. MNPERE, 2003.

[4] Wikipedia. Torque. *INTERNET LINK*, -.

Appendices

A Table of quantities and units

Table 2: The electrical quantities

Symbol	Quantity	Unit
C	Capacitance	F
i	Instantaneous current	A
$\hat{I}$	Amplitude of current	A
$\bar{I}$	Current phasor	A
I	RMS value current	A
j	imaginairy unit	$-$
L	Inductance	H
p	Instantaneous power	W
P	Real power	W
Q	Reactive power	$V\ A\ R$
R	Resistance	Ω
S	Complex power	$V\ A$
T_s	Time period of controller	s
dT_s	Time control output is 1	s
v	Instantaneous voltage	V
$\bar{v}$	Average instantaneous voltage	V
$\hat{V}$	Amplitude of voltage	V
$\bar{V}$	Voltage phasor	V
V	RMS value voltage	V
X	Impedance magnitude of component	Ω
Y	Admittance	S
Z	Impedance	Ω
φ	Angle in complex plane	rad
ω	Rotational velocity	$\frac{rad}{s}$
ψ	Flux linkage	$Wb\,t$

Table 3: The mechanical quantities

Symbol	Quantity	Unit
B	Rotational damping	$\frac{N\ m\ s}{rad}$
E	Energy	J
I	Area moment of inertia	m^4
J	Moment of inertia	$kg\,m^2$
K	Rotational spring	$\frac{N\ m}{rad}$
p	Power	W
r	Arm of gear	m
T	Torque	$N\ m$
t	Time	s
W	Work	$N\ m$
θ	Angle	rad
ω	Rotational velocity	$\frac{rad}{s}$
$\dot{\omega}$	Rotational acceleration	$\frac{rad}{s^2}$
τ	Time (used for integration)	s